MATHEMATICAL MODELLING OF FLUIDIZED BED COAL COMBUSTION

I. HYDRODYNAMICS

Report by
H.A. Becker, R.K. Code, and P.E.G. Gogolek
Department of Chemical Engineering
Queen's University at Kingston
Kingston, Canada

Technical Report QFBC.TR.89.2
Queen's Fluidized Bed Combustion Laboratory

Work on this project was conducted under the auspices of the Canada Centre for Mineral and Energy Technology, Energy, Mines, and Resources Canada

DSS File Number:	24ST.23440-7-9015
DSS Contract Serial Number:	OST84-00512
CANMET Scientific Authority:	Dr. E.J. Anthony
DSS Science Contracting Officer:	Ms. M. Ouellet

47 pp

MICROMEDIA

91-01583/

Abstract

Modelling hydrodynamics is the first step in modelling fluidized bed combustion. Regions of characteristic hydrodynamics are identified and delimited according to the particle concentration profile. This is the basis for the Three Zone model, these zones being the Dense Bed, the Splash Zone and the Freeboard. The effect of the presence of tubes on the extents of the zones is included. The two-phase flow of gas in the Dense Bed is examined in some detail. The flow of particles between zones and in the Freeboard is discussed. There are several appendices giving modelling details and supporting evidence for some modelling decisions.

Resumé

Établir un modèle de l'aspect hydrodynamique est la première étape de l'établissement d'un modèle de la combustion à lit fluidisé. Les régions de conditions hydrodynamiques caractéristiques sont identifiées. Ces régions sont délimitées selon les profils de concentration des particules solides. Ceci appuie le modèle des trois régions («Three zone model»), ces régions étant dénommées le lit dense («Dense Bed»), la région d'explosion des bulles («Splash Zone») et la région de transport des particules solides («Freeboard»), respectivement. On inclut l'influence de la présence de tuyaux sur les dimensions des régions. On examine en détails la circulation du gaz en deux phases dans le lit dense. On examine aussi la circulation des particules solides d'une région à l'autre et dans la région du «Freeboard». Plusieurs annexes sont incluses dans lesquelles on donne les détails de modèlisation avec des justifications appuyant certains décisions.

Nomenclature

Variables

A_c	cross sectional area of the column, m^2
A_D	catchment area of bubble at the distributor, m^2
c	molar concentration, $kmol/m^3$
c_A	molar concentration of species A, $kmol/m^3$
d_b	bubble diameter, m
d_c	column diameter, m
d_{orf}	distributor plate orifice diameter, m
d_p	particle diameter, m
d_t	tube diameter, m
D	coefficient of molecular diffusion, m^2/s
D_T	coefficient of mechanical or turbulent diffusion, m^2/s
g	acceleration due to gravity, m/s^2
G	mass flux density, kg/m^2s
h	enthalpy per unit mass, J/kg
H	enthalpy, J
H_A	molar enthalpy of species A, J/kmol
K_{SZ}	particle concentration decay parameter, m^{-1}
L	length, m
m	total mass, kg
M_A	molar mass of species A, kg/kmol
n	total moles, kmol
N	total number
p	total pressure, $N/m^2 = Pa$

p_A	partial pressure of species A, $N/m^2 = Pa$
t	time, s
T	temperature, K, or length of time interval, s
u	local gas velocity field, m/s
u_p	local particle velocity field, m/s
U	mass transport average velocity field, m/s
V	total volume, m^3
$\dot{V}$	volumetric flux, m^3/s
w_A	mass fraction of species A
x_A	mole fraction of species A
Y	adjustment factor for two-phase flow, $0 < Y \leq 1$
z	vertical position, m
β_d	fraction of particles falling, m
δ	boundary layer thickness, m
ϵ	volume fraction of fluid or fraction void
ϵ_b	volume fraction of bubbles
ϵ_p	volume fraction of particles, $= 1 - \epsilon$
λ_p	ratio of particle spacing to particle diameter
μ	dynamic viscosity, kg/ms
ξ	proportionality constant for particle transport by bubbles
ρ	mass density of fluid, kg/m^3
ρ_p	mass density of particles, kg/m^3
$\Delta\rho$	density difference, $= \rho_p - \rho$, kg/m^3
θ	mean residence time, s
τ	dummy time variable for integration or period of cycle, s
Φ	mass feedrate of particles per unit area, kg/m^2s
ψ	particle sphericity

Subscripts

o	reference, initial or slumped value
A	species *A*
b	bubble or bubble phase
bed	bed
c	column
chk	at the point of choking
d	down
DB	Dense Bed region
ex	linearly extrapolated value
f	fuel
F	Freeboard region
i	i^{th} interval of partition of particle size range
mb	at the point of minimum bubbling
mf	at the point of minimum fluidization
n	counting index
p	particle or particle phase
s	stone
SZ	Splash Zone region
t	tube or tube bundle

Superscripts

(*b*)	occupied by bubbles
(*e*)	occupied by emulsion

Dimensionless Parameters

Ar	Archimedes number, $\frac{d_p^3 g(\rho_p - \rho)\rho}{\mu^2}$
d_p^*	Grace's dimensionless particle diameter, $Ar^{1/3}$
Fr_c	column Froude number, $\frac{(U - U_{mb})^2}{g d_c}$
Fr_{DB}	Dense Bed Froude number, $\frac{(U - U_{mb})^2}{g(0.5 L_{DB} + 4\sqrt{A_D})}$
Fr_p	particle Froude number, $\frac{U^2 \rho}{g(\rho_p - \rho) d_p}$
Re_{mb}	particle Reynolds number at minimum bubbling, $\frac{\rho U_{mb} d_p}{\mu}$
U^*	Grace's dimensionless velocity, $U\left[\frac{\rho^2}{g(\rho_p - \rho)\mu}\right]^{1/3}$

Table of Contents

	Page
Abstract	ii
Nomenclature	iii
Table of Contents	vii
Introduction	1
1. The Three Zone Model	2
2. Gas Flow	6
3. Particle Flow	11
4. Conclusions and Recommendations	13
Appendices	
A. Dimensionless Groups and Particle Classification	14
B. Bubble Fraction Formula	16
C. Adjustment of Interstitial Flow due to Bubbles	18
D. Bubble Growth Formulae	20
E. Fluidization Dynamics and Modelling of AFBC	22
References	26
Figures	29

Introduction

In the mathematical modelling of fluidized bed combustion the hydrodynamics have a supportive role. Of primary interest is the chemical and thermal production of the unit. The hydrodynamics have a scientific interest, of course, but the details required must be evaluated against the effect on chemical and thermal processes. However, it has been shown on numerous occasions that the peculiar hydrodynamics of fluidized beds have great influence on chemical processes. Because chemical processes have primacy, some properly hydrodynamic phenomena will be discussed in another report on reaction modelling.

The gas and particle flows are all modelled in one dimension and at steady state. Thus the quantities like voidage and velocity are averages over the cross-section and time. This averaging loses some information and care must be taken not to neglect important terms in the averaged equations. The one-dimensional hydrodynamics of bubbling have been extensively studied. Similarly, there is some understanding of the transport of dilute suspensions of particles by a gas stream. The bubbling occurs at the bottom of the column, while the dilute transport occurs at the top. This separation is the motivation for the division of the column into three zones, isolating the transition zone which is poorly understood. The usual approach has been to use two zones, extending the bubbling and transport zones beyond the regions of applicability.

We have written about the bubbling bed, but because there may not be bubbles as such this is more properly called the dense bed. When the bottom region has a large number of fixed cooling tubes, the behaviour is markedly different. The tubes act to break up bubbles and restrict the movement of particles. This produces a more homogeneous mixture of gas and particles with a great deal of seemingly random lateral motion. This phenomenon has been called tube-ulence,[1] because it resembles turbulence and is caused by the tubes. We call a tube-ulent bed a constrained bed, in contrast with a free (freely bubbling) bed. The three zones have different extents depending on whether the bed is free or constrained.

We have tried to use mechanistic descriptions of the phenomena as far as possible and avoided empirical descriptions for two reasons. First, more insight is gained from mechanism. Second, purely empirical relations have a tendency to fail badly outside the range of experiments, or even on a different unit. This brings up the problem of scaling and dimensionless parameters for fluidized bed hydrodynamics, which is discussed briefly in Appendix A. There are three length scales, corresponding to the particle size, the bubble size (or tube spacing), and the column diameter, which are called the micro-, meso- and macro-scales. Sometimes the meso- and macro-scales are the same, for example in a slugging bed. The micro-scale is important for transition phenomena like the onset of fluidization. The meso-scale is important in the actual flows, such as bubble rise and coalescence. It is not expected that the micro-scale influences meso-scale phenomena, except perhaps in a way analogous to a material property, like viscosity. This dichotomy makes hydrodynamic modelling easier, since the micro-scale can often be neglected; the best example being the success of the bubble growth formula of Mori and Wen (see Appendix D).

The first section deals with the details of calculating the three zones for both a bubbling and a tube-filled bed. This is followed by a section on gas flow which is mostly concerned with some revisions to standard two-phase analysis of bubbling beds. The third section deals with the flow of particles. Its brevity is due to the placement of the discussion of population balance in the reaction modelling report. There are also five supporting appendices.

---

1 We first heard this term from Sven Andersson of Chalmers University at the Amsterdam IEA Mathematical Modelling meeting in November, 1988.

1. The Three Zone Model

In the mechanics of gas-solids flow there is a fair understanding of the processes in a bubbling fluidized bed and the dilute transport of particles. Both these phenomena are found in a fluidized bed combustor, the former at the bottom of the column, the latter at the top. Between the two there is a significant and poorly defined transition zone, sometimes called the splash zone. Because of the large number of diverse phenomena which occur in the transition region, a mechanical description cannot be expected. The idea behind the explicit introduction of this transition zone is to isolate the uncertainty, allowing confident modelling of the other two regions. Hopefully some reasoned guesses will do an adequate job with the transition zone. This separation also shows the area where improvement can be sought most profitably.

The three zones are the Dense Bed (DB), the Splash Zone (SZ), and the Freeboard (F). These terms are used with some trepidation given the preconceptions associated with each. The boundaries between the zones will be defined and calculated in terms of the assumed voidage profile in each; the calculations are of importance, not the names assigned. All flows are assumed one-dimensional and steady-state in a column of uniform cross-section. The details of the flows are given in later sections, with the incorporation of chemical reaction to follow in another report.

The three zones are illustrated in Figure 1.1. The Dense Bed zone extends from the distributor for a height L_{DB} and has the following properties:

1. Constant average voidage ϵ_{DB} over height
2. Lowest average voidage in the column
3. Uniform particle composition
4. Every point equally accessible to every particle
5. Without tubes, the gas flow split between bubbles and emulsion
6. With sufficient tubes, uniform gas-solid mixture
7. Isothermal

The Splash Zone extends from the top of the Dense Bed for some height L_{SZ}. It is worthwhile considering some of the phenomena occurring in this transition zone before listing the modelling assumptions. As an isolated bubble rises to the top of a fluidized bed it pushes up a dome of particles. This dome bursts, ejecting particles and a rush of gas. For this case, a variety of estimates have been made of particle trajectories, based on a variety of assumptions. In a vigourously bubbling bed, such as used for FBC, there is no well-defined top of the bed. There are interactions between bubbles, like coalescence and the heaving of the bed. The bursts of gas interact with each other, with the ejected particles from other bubbles, and with the descending particles. The ejected particles interact with each other, with those ejected from other bubbles and with descending particles. In the region where the particle concentrations are high these interactions must be significant. Trajectory calculations necessarily ignore them and therefore can not be suitable for that region. With these observations in mind the following assumptions are made:

1. Continuous particle concentration within the zone
2. Continuous particle concentration across the boundaries
3. Exponentially decreasing particle concentration with height above L_{DB}

4. Particle accessibility to any point proportional to the average particle concentration

5. Uniform particle composition

6. Plug flow of gas in the continuous phase

7. Isothermal at the same temperature as the Dense Bed

Some of these assumptions, particularly (4) and (5), are inadequate, with the only justification being simplicity.

The Freeboard region starts where the particle concentration is low enough for dilute transport and extends to the top of the column. The modelling assumptions are:

1. Plus flow of gas and particles

2. Particles either ascending or descending at an average velocity

3. Variable temperature

4. Gas and particles at the same temperature

The ascending and descending streams can be either of an annular structure or as counter-flowing streams in the same region.

From these assumptions the following defining relationships hold. Consider the voidage function $\epsilon(z)$, $0 \leq z \leq L_c$, which is

$$\epsilon(z) = \begin{cases} \epsilon_{DB}, & 0 \leq z \leq L_{DB} \\ \epsilon_{SZ}(z), & L_{DB} \leq z \leq L_{DB} + L_{SZ} \\ \epsilon_F(z), & L_{SZ} + L_{DB} \leq z \leq L_c \end{cases} \tag{1.1}$$

The exponential profile in the Splash Zone has

$$\epsilon_{SZ}(z) = 1 - (1 - \epsilon_{DB})e^{-K_{SZ}(z - L_{DB})}, \tag{1.2}$$

incorporating continuity at the boundary between the Dense Bed and the Splash Zone. Denoting the minimum voidage for dilute transport by ϵ_{Fo}, continuity at the top of the Splash Zone gives

$$\epsilon_{Fo} = 1 - (1 - \epsilon_{DB})e^{-K_{SZ}L_{SZ}} \tag{1.3}.$$

Let ΔP be the static pressure drop along the full length of the column. Assuming this drop is due solely to the weight of particles in the column

$$-\frac{\Delta P}{g} = (1 - \epsilon_{DB})\rho_{pDB}L_{DB} + \rho_{pSZ}\int_{L_{DB}}^{L_{DB}+L_{SZ}}[1 - \epsilon_{SZ}(z)]dz + \int_{L_{DB}+L_{SZ}}^{L_c}\rho_p(z)[1 - \epsilon_F(z)]dz. \tag{1.4}$$

The integral along the Freeboard usually can be neglected. Assuming $\rho_{pDB} = \rho_{pSZ} = \rho_p$, this gives

$$-\frac{\Delta P}{g\rho_p} = (1 - \epsilon_{DB})L_{DB} + \int_{L_{DB}}^{L_{DB}+L_{SZ}}[1 - \epsilon_{SZ}(z)]dz \tag{1.5}$$

and this also gives relations for the slumped bed, the bed at minimum bubbling and the extrapolated bed height.

$$-\frac{\Delta P}{g\rho_p} = (1-\epsilon_o)L_o = (1-\epsilon_{mb})L_{mb} = (1-\epsilon_{DB})L_{ex} \tag{1.6}$$

Pressure drop, because it is a measurable operating parameter, has been related to the weight of particles in the column. A more direct measurement, for example L_o, is more difficult to obtain; if it is available then (1.6) can be used to calculate ΔP. Putting (1.2) into (1.5) gives

$$-\frac{\Delta P}{g\rho_p} = (1-\epsilon_{DB})L_{DB} + \left[\frac{1-\epsilon_{DB}}{K_{SZ}}\right](1-e^{-K_{SZ}L_{SZ}})$$

and (1.3) gives

$$-\frac{\Delta P}{g\rho_p} = (1-\epsilon_{DB})L_{DB} + \left[\frac{\epsilon_{Fo}-\epsilon_{DB}}{K_{SZ}}\right] \tag{1.7}.$$

These equations are necessary consequences (within certain approximations) of the modelling assumptions for the three zones. The unknowns are L_{DB}, L_{SZ}, ϵ_{DB} , K_{SZ} and ϵ_{Fo} and there are only two independent relations, (1.3) and (1.7). Further relations, either empirical or constitutive, must be introduced.

The Dense Bed voidage can be estimated readily from pressure drop measurements. Such measurements should yield a linear change with height, or can be approximated by a line, and the slope gives ϵ_{DB}. An alternative method uses some modification of two-phase theory (see the next section) to calculate the bubble fraction. The following formula is used

$$\epsilon_b = \frac{0.3Fr_{DB}^{\ 0.4}}{Fr_{DB}^{\ 0.4}+0.52} \tag{1.8}$$

where

$$Fr_{DB} = \frac{(U-U_{mb})^2}{g(0.5L_{mb}+4\sqrt{A_D})} \tag{1.9}$$

is a Dense Bed Froude number. A rationale for this relation is given in Appendix B. The Dense Bed voidage is given by

$$\epsilon_{DB} = \epsilon_b + (1-\epsilon_b)\,\epsilon_{mb} \tag{1.10}.$$

The minimum bubbling velocity is calculated from

$$Re_{mb} = [C_1^2 + C_2 Ar]^{1/2} - C_1 \tag{1.11}$$

where $C_1 = 25.28$ and $C_2 = 0.0571$. This relationship was originally formulated for uniform particles, but Farkas [1988] found that it also works for particles with a wide size distribution, using the reciprocal mass average as the representative particle diameter. The voidage at minimum bubbling is given by the Broadhurst and Becker [1973] correlation

$$\epsilon_{mb} = 0.49\,\psi^{-0.72}Ar^{-0.029} \tag{1.12}$$

approximating the density ratio term by 0.83. Knowing ϵ_{mb} allows us to calculate L_{mb} from (1.6).

The system of equations is closed by setting the dilute transport voidage and the Dense Bed height. The minimum voidage for dilute transport can be estimated by bounding the mean spacing among particles. Let λ_p be the ratio of mean particle spacing to some mean particle diameter: a value of λ_p around 3 or 4 should be sufficient for dilute transport. A simple cell model gives

$$\epsilon_{Fo} = 1 - \lambda_p^{-3} \tag{1.15}$$

as the voidage at the start of the Freeboard.

Andersson [1988] found that $L_{DB} = L_{mb}$ for freely bubbling beds at velocities below 3 m/s. From (1.7) we get

$$K_{SZ} = \frac{\epsilon_{Fo} - \epsilon_{DB}}{(\epsilon_{DB} - \epsilon_{mb})L_{mb}} \quad . \tag{1.16}$$

Equation (1.3) gives

$$L_{SZ} = \frac{1}{K_{SZ}} \log\left[\frac{1 - \epsilon_{DB}}{1 - \epsilon_{Fo}} \right] . \tag{1.17}$$

This fully defines the three zones and the average voidages in each. The behaviour of these values as a function of the excess gas velocity is shown in Figures 1.2 and 1.3.

For a tube-filled bed the voidage profile is different. There is an expansion of the Dense Bed region and a more rapid drop in the Splash Zone particle concentration because the bubble size is restricted. The relations (1.3), (1.7) and (1.15) remain valid. The voidage correlation of Canada et al. [1978] is used,

$$\epsilon_{DB} = \frac{U \epsilon_{mb}}{1.05\, \epsilon_{mb} U + (1 - \epsilon_{mb}) U_{mb}} \quad . \tag{1.18}$$

The length scale for the exponential decay in the Splash Zone is related to the bubble size at the top of the Dense Bed. The tube spacing ΔL_t limits the bubble growth, thus

$$K_{SZ} = \frac{1}{\Delta L_t} \tag{1.19}$$

is a reasonable estimate.

With these estimates, (1.7) gives the extent of the Dense Bed

$$L_{DB} = \frac{(1 - \epsilon_{mb})L_{mb} - (\epsilon_{Fo} - \epsilon_{DB})\Delta L_t}{1 - \epsilon_{DB}} \tag{1.20}.$$

The extent of the Splash Zone comes from (1.17). The average voidage in the Splash Zone is

$$\bar{\epsilon}_{SZ} = 1 - \frac{\epsilon_{Fo} - \epsilon_{DB}}{K_{SZ} L_{SZ}} \tag{1.21}.$$

The results are shown in Figures 1.4 and 1.5 for $\Delta L_t = 0.08$m .

2. Gas Flow

It is necessary first to consider the magnitude of the effect of chemical reaction and particles flows on the gas flow. The primary influence of chemical reaction on gas flow is the change in density due to the change in composition. The temperature effect is negligible because of the high heat transfer rates to bed particles. In combustion the major contributor to composition change is the replacement of oxygen by combustion products like carbon dioxide, sulphur dioxide, nitric oxide and water. A simple calculation shows that total replacement will increase the gas density by about 10%. Release of carbon dioxide from the calcination of limestone and water vapour from all solids fed will increase the mass flowrate of gas. The total increase can be estimated by

$$\frac{[(1-w_{H_2O})(1-w_{ash})+w_{H_2O}]\Phi_f}{\rho_o U_o}$$

for the fuel, and

$$\frac{[0.44(1-w_{H_2O})w_{CaCO_3}+w_{H_2O}]\Phi_s}{\rho_o U_o}$$

for the stone, where the Φ are feedrates per unit area. This should be less than 10%. Similarly, the net particle velocity between the feed and removal points is estimated as

$$\frac{\Phi_f}{\rho_{pf}}+\frac{\Phi_s}{\rho_{ps}}$$

and this is small compared to the superficial gas velocity and the interstitial gas velocity for bubbling beds. For circulating beds the high recirculation rates means the net particle velocity needs to be considered in the analysis of the dense region. However, for bubbling beds, the reaction effect and the net solids flow can be neglected.

Consider first the free bed, illustrated in Figure 2.1. Here the bubbles are distinct and persistent and therefore must be explicitly included in the flow analysis. Following Grace and Clift [1974], we will look at the time averaged material balances in the bed. The gas and particles are considered incompressible and steady-state has been attained.

Let ϵ, u and u_p be the point functions for the local voidage, vertical gas velocity relative to the particles, and vertical particle velocity respectively. Assume $\epsilon = 1$ and $u_p = 0$ in a bubble. Define $A_c^{(b)}$ as the subset of A_c occupied by bubbles and $A_c^{(e)}$ as the subset of A_c occupied by emulsion,[2] so that

$$A_c = A_c^{(b)} \cup A_c^{(e)} \ .$$

Although A_c is unchanging, $A_c^{(b)}$ and $A_c^{(e)}$ vary with time and vertical position. Define the time average of a function χ as

$$\bar{\chi}(z) = \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} \chi(z,\tau)d\tau = \lim_{T\to\infty} \frac{1}{T}\int_0^{T} \chi(z,\tau)d\tau \qquad (2.1)$$

the two definitions being equivalent under steady-state conditions. The bubble fraction is

---

[2] These symbols will be used both as sets and as numerical values of the areas involved. This abuse of notation will hopefully not cause much confusion though it does obscure some of the subtleties of the averaging process.

$$\epsilon_b = \frac{\overline{A_c^{(b)}}}{A_c} \tag{2.2}$$

and immediately

$$\overline{A_c^{(e)}} = (1 - \epsilon_b) A_c \ . \tag{2.3}$$

Consider first a time-averaged particle volume balance over the region contained within the column between the distributor and the plane $z = z_1$:

$$\overline{\int_{A_c} u_p (1 - \epsilon) dA} = \overline{\int_{A_c^{(e)}} u_p (1 - \epsilon) dA} = 0 \tag{2.4}$$

because there is no net particle flow. A time-averaged total volume balance over the same region gives

$$UA_c = \overline{\int_{A_c^{(b)}} u dA} + \overline{\int_{A_c^{(e)}} [(u_p + u)\,\epsilon + u_p (1 - \epsilon)] dA} \tag{2.5}$$

Since $A_c^{(b)}$ is composed of separate regions it is reasonable to replace the first integral with a sum over the cut areas of the bubbles. Approximating the gas velocity across these sections with a single value for each bubble gives

$$\overline{\int_{A_c^{(b)}} u dA} = \overline{\sum_n U_n^{(b)} A_n} \tag{2.6}$$

where

$$A_c^{(b)} = \sum_n A_n \ .$$

This gives the basic formulation for two-phase flow[3]

$$UA_c = \overline{\sum_n U_n^{(b)} A_n} + \overline{\int_{A_c^{(e)}} u_p dA} + \overline{\int_{A_c^{(e)}} u\,\epsilon dA} \tag{2.7}$$

The first assumption of two-phase theory is constant voidage in the emulsion phase, i.e. $\epsilon = \epsilon^{(e)} < 1$ in $A_c^{(e)}$. It follows immediately from (2.4) that

$$\overline{\int_{A_c^{(e)}} u_p dA} = 0 \ .$$

The assumption seems reasonable despite some experimental evidence of slightly increased emulsion voidage due to bubble passage. It is further assumed that $\epsilon^{(e)} = \epsilon_{mb}$.

The gas flow in the emulsion is also set to be a constant, the value at minimum bubbling, in standard two-phase theory. This cannot be so. The gas flow relative to the particles is driven by the local pressure gradient, which changes in the neighbourhood of a bubble. Thus u and $A_c^{(e)}$ do not vary independently. A calculation of the covariance would be much too difficult

---

[3] This equation assumes constant flow through the distributor. In the case of very large pressure fluctuations in the bed it is possible that there is feedback to the windbox. This may be happening in the apparent slugging regime.

so an estimate of the effect of bubble fraction on the interstitial flow was made, and is found in Appendix C. The result shows the average interstitial velocity is

$$U^{(e)} = \frac{U_{mb}}{(1-\epsilon_b{}^{2/3})} \tag{2.8}$$

and so

$$\overline{\int_{A_c{}^{(e)}} u\,\epsilon dA} = U^{(e)}(1-\epsilon_b)A_c = U_{mb}\,A_c\left[\frac{1-\epsilon_b}{1-\epsilon_b{}^{2/3}}\right] \tag{2.9}$$

Returning to (2.7) gives the bubble through flow velocity as

$$U^{(b)} = \frac{1}{\epsilon_b A_c}\overline{\sum_n U_n{}^{(b)} A_n} = \frac{U}{\epsilon_b} - \frac{U_{mb}}{\epsilon_b}\left[\frac{1-\epsilon_b}{1-\epsilon_b{}^{2/3}}\right] \tag{2.10}$$

Note that $U^{(b)}$ is the velocity, relative to the column, of the gas in the bubble. In this derivation, no use has been made of empirical bubble properties like bubble rise velocity or bubble diameter. The only conditions are from the derivation of (2.8) where gas is assumed not to recirculate to the bubble and follow the Davidson-Harrison volumetric through flow relation.[4] Since the bubble fraction is constant in the Dense Bed, the through flow velocity is also constant. Thus the two-phase analysis has given the split of the gas flow between the bed and the emulsion.[5]

There have been several efforts to produce a formula for bubble size as a function of excess gas velocity, column diameter, distributor design and position in the bed. Appendix D compares three of these and shows that the formula of Mori and Wen [1975] is good for a limited data set, from cold and hot beds of large particles. This is interesting because the formula was originally fitted to data from small-particle beds. This is further evidence that the meso-scale phenomena, like bubbling, is nearly independent of particle properties.

An objection has been made to the formula of Mori and Wen that the final, asymptotic, bubble size can be smaller than the initial bubble size. This would have the bubbles decrease in size with height. For a porous plate, $d_{b\infty} < d_{bo}$ when

$$1.49[d_c{}^2(U-U_{mb})]^{0.4} < 3.69\,\frac{(U-U_{mb})^2}{g} \tag{2.11}$$

or

$$Fr_c = \frac{(U-U_{mb})^2}{g d_c} > 0.569 \tag{2.11}$$

This comes from (D.2) and (D.3). However, for an excess gas velocity in this range the initial bubble size from (D.3) is

$$d_{bo} > 2.1 d_c\ .$$

---

4 It may appear that the development of (1.8) for bubble fraction uses a form of the two-phase theory inconsistent with this one. We prefer to consider (1.8) as an empirical relation with the parameter Y fitted to the experimental data. This view obviates the inconsistency.

5 Neglecting the covariance terms in the averaged equations can be dangerous. Birkhoff [1964] provides an example of two-phase flow where the covariance term is the largest one. Calculating the covariances would require changing the averaging process from a time average (2.1) to an ensemble average.

Hence the bubbles are initially in the slugging regime and the bubble growth formula does not apply nor is it needed.

The special hydrodynamics of initially slugging beds is being investigated. Preliminary ideas have been advanced by Becker and Gogolek [1988] but more work needs to be done.[6] The two-phase analysis above should still apply, but it does not take full advantage of the special structure of the flows.

The initially slugging behaviour should not be confused with the so-called 'exploding bubble' regime proposed by Fitzgerald [1985]. This regime is characterized by bubbles which grow at least as fast as they rise in the bed,

$$\frac{d}{dt} d_b \gtrsim u_b$$

which is equivalent to

$$\frac{d}{dz} d_b \gtrsim 1 \ . \tag{2.12}$$

The analysis procedes as follows. The correlation of Cranfield and Geldart [1974] for bubble growth in large-particle beds is

$$d_b = 2.26(U - U_{mf})^{1.11} z^{0.81} \tag{2.13}$$

The regime boundary, using (2.12), is[7]

$$U - U_{mf} = 26.4 z^{0.17} \tag{2.14}$$

The bed is in the exploding bubble regime when

$$U - U_{mf} = 26.4 L_{bed}^{\ 0.17} \ . \tag{2.15}$$

The following objections must be raised. First, the correlation of Cranfield and Geldart has been shown to severely overpredict bubble sizes at large excess gas velocities (Glicksman et al. [1987]). Second, the choice of measure of bed height, L_{bed}, is not made clear. Third, if (2.12) is a local condition, i.e. a bubble satisfying (2.12) is 'exploding', then (2.14) should really be written as

$$z \leq 4.93 \times 10^{-9} (U - U_{mf})^{5.86} \tag{2.16}$$

so the bottom part of every bed is in the exploding bubble regime. Equation (2.15) requires every bubble to be exploding, so that 90% of the bed can be satisfy (2.12) but still not be in the exploding regime. This is clearly unreasonable.

If we try to repeat the analysis of Fitzgerald with the more accurate formula of Mori and Wen, we get

$$\frac{0.3(d_{b\infty} - d_{bo})}{d_c} e^{-0.3z/d_c} \gtrsim 1 \tag{2.17}$$

This can be written as

$$0.3(2.35 Fr_c^{\ 0.2} - 3.69 Fr_c) e^{-0.3z/d_c} \gtrsim 1$$

---

6 This paper is reproduced in Appendix E.

7 Fitzgerald uses 21.85 in place of 26.4, but that shows an error in differentiation.

There are no positive values of Fr_c which satisfy this inequality and so there are no exploding bubbles.

The transfer rate between the bubble and emulsion is needed to complete the two-phase analysis of gas flow. This is more coherently discussed in the context of reaction modelling and will be found in that report. The region close to the distributor does not have distinct bubbles, and the phenomena are not clearly understood. Some investigators claim there are gas jets which entrain particles and eventually form bubbles; others claim a succession of bubbles are blown with a region of increased voidage around them. There is a concensus however that gas and particles come in close contact in that region and a strong effect on conversion is noted. Perhaps a plug flow model is appropriate here; this will be investigated in the report on reaction modelling.

For a constrained bed the bubbles are subjected to repeated splitting and coalescence and there is a great deal of mixing with the particles. Two-phase analysis is inappropriate because of the transient nature of the bubbles. Plug flow is a reasonable approximation for the gas flow in this situation.

For both the Splash Zone and the Freeboard the gas is modelled as plug flow in one phase.

3. Particle Flows

As stated in section 1, the Dense Bed and the Splash Zone are modelled as well-mixed for the particles. This may cause some confusion. A reactor is well-mixed if there is compositional uniformity throughout its volume. In this case, the volume is not the volume of the zone but the volume of the particles in the zone. In particular, this volume varies with height in the Splash Zone. Thus the particles in either zone show no segregation and no variation of composition. This assumption is common for the Dense Bed zone, due to the mixing by bubbles, and seems to be valid when $U/U_{mb} > 5$ (Farkas [1988]). In the Splash Zone there is a high circulation rate of particles. Splashing and heaving caused by bursting bubbles produces particle clusters of varying size and velocity. These rise and fall, collide, disintegrate and reform. While it is hard to argue against segregation in the Splash Zone, it is equally hard to predict such segregation. The high circulation rate of particles, with a relatively high particle concentration, acts to maintain the Splash Zone temperature close to that of the Dense Bed. Some experimental evidence for this assumption can be found in Gõrmar et al. [1989], especially their Figure 3 which shows a constant temperature region extending well past the Dense Bed. They are measuring temperature in a tube-filled region, so this is direct support of the assumption for constrained beds. For a free bed the Splash Zone has much greater extent and there may be a temperature change towards the top, where the particle concentration is low.

With the assumption of well-mixedness, the details of particle flows within the Dense Bed and the Splash Zone can be ignored.[8] The particle flow in the Freeboard is modelled as counter-current plug flow with transfer only from the upflowing stream to the downflowing stream. We had originally considered this transfer mechanism to be turbulent dispersion to a film along the walls. We have since been convinced that this is incorrect. The form of the flow equations does not change, only the particulars of the transfer term. The transfer term is the method for calculating the change in the particle size distribution along the Freeboard. Larger particles are preferentially transferred to the downflowing stream. The upward velocity is taken to be the same for all particles, regardless of size or density. this velocity will change if the gas flow changes, for example by a change in temperature. The amount of change is such as to keep the slip velocity of the particles constant. The initial condition for particle concentration is given by (1.15). The initial particle velocity is thus calculated from the particle mass flux from the Splash Zone to the Freeboard.

The particle flow from the Splash Zone to the Freeboard should be a function of the particle size distribution in the Splash Zone. An approach using a linear dependence on particle size fraction and choking correlations for particles of a single size is described in Gogolek [1988]. For particles of a single type, this method is as follows. Let $G_{p\,chk\,i}$ be the mass flux at choking conditions for particles of size i. The total particle flux is

$$G_p = \sum_i x_{SZ\,i} G_{p\,chk\,i} \tag{3.1}$$

and the size distribution in the particle stream is

$$x_{Fo\,i} = \frac{x_{SZ\,i} G_{p\,chk\,i}}{G_p} \tag{3.2}$$

The initial particle velocity in the Freeboard is thus

---

8 Except for plume model calculations but those will be discussed in the reaction modelling report.

$$U_{po} = \frac{G_p}{\rho_p(1-\epsilon_{Fo})} \tag{3.3}$$

Unfortunately this method seems to underpredict the particle flux. It possibly needs to incorporate some features of a particle trajectory model.

The particle flow from the Dense Bed to the Splash Zone is driven by the passage of bubbles. Each bubble pushes particles into the Splash Zone. Bubbles that coalesce as they cross the boundary transfer particles at high velocity. The volumetric flux of bubbles is given by

$$\dot{V}_b = Y(U - U_{mb})A_c$$

with $Y = 0.3$ as shown in Appendix B. The particle volumetric flux is assumed to be proportional to $\dot{V}_b$, thus

$$\dot{V}_p = \xi\,(1-\epsilon_{mb})Y(U - U_{mb})A_c \tag{3.4}$$

and[9] $\xi = 0.16$. For a constrained bed the mechanisms are different, because bubbles do not exist. The mixing model of Staub [1979] can be adapted to calculate the particle transfer flux. From his full model Staub fit a curve for the superficial upflowing particle velocity. We have explicitly placed the tube spacing in this expression to get the following formula for the volumetric flux of particles.

$$\dot{V}_p = 0.234(1-\epsilon_{DB})\sqrt{g\Delta L_t}\,A_c \tag{3.5}$$

Thus (3.4) or (3.5) gives the particle flow from the Dense Bed to the Splash Zone.

The reflux from the Freeboard to the Splash Zone comes from the Freeboard flow model and is the integral of the transfer term over the length of the Freeboard. The reflux from the Splash Zone to the Dense Bed is a calculated parameter weighted by the difference between the bubble transfer rate (3.4) and the choking transfer rate. The method of calculation is described in the population balance section of the reaction report.

The particle flow through the cyclone is modelled by a single capture efficiency parameter, the fraction of particle flow from the Freeboard captured by the cyclone. This should be improved to accomodate a measure of the variation of efficiency with particle size, such as a cut size.

---

9 This value comes from Geldart [1985] who uses $Y = 0.5$ and $\xi = 0.1$.

4. Conclusions and Recommendations

The hydrodynamics of fluidized bed combustors have been described within the framework of a Three Zone Model. We have shown how to calculate the zones for both free and constrained beds. These calculations still need to be tested experimentally, especially for constrained beds. Furthermore, these are only two extreme cases and it remains to show what to do for intermediate situations.

The gas flow in the Dense Bed zone is not affected by either chemical reaction or net solids flow. Our revised two-phase analysis gives new estimates for gas velocity in the emulsion and bubbles. The formula of Mori and Wen for bubble size is found to be suitable for FBC. In a constrained bed, in the Splash Zone, and in the Freeboard the gas is in plug flow. The special hydrodynamics of initially slugging beds should be more deeply investigated.

The particles in the Dense Bed and the Splash Zone are well-mixed. In the Freeboard the particles have two counter-flowing streams, with the segregation produced by the transfer between streams. The particle flows between the zones are given. The particle flux from the Splash Zone to the Freeboard needs to be improved, but it is not clear how to do this. Finally, a new cyclone description incorporating particle size effects is needed.

A. Dimensionless Parameters and Particle Classification

In hydrodynamics a central role in the description of phenomena is played by certain dimensionless parameters. These are formed from products and ratios of characteristic values of the variables of the particular phenomenon in question. Broadhurst and Becker [1973] selected the following variables as important for fluidization:

$$d_p\ ,\ g(\Delta\rho)\ ,\ \rho\ ,\ \mu\ ,\ U\ ,\ \rho_p\ ,\ \psi\ ,\ d_c\ ,\ L_0;$$

and formed the following set of six dimensionless parameters:

$$\frac{{d_p}^3 g(\rho_p-\rho)\rho}{\mu^2} = Ar\ ,\ \frac{U^2\rho}{g(\rho_p-\rho)d_p} = Fr_p\ ,\ \frac{\rho_p}{\rho}\ ,\ \psi\ ,\ \frac{d_p}{d_c}\ ,\ \frac{d_p}{L_0}\ .$$

Grace [1986], in a review of gas-solid mechanics, makes extensive use of two parameters

$$d_p^{\ *} = Ar^{1/3},\quad \text{and}\quad U^* = U\left[\frac{\rho^2}{g(\rho_p-\rho)\mu}\right]^{1/3} = (Fr_p d_p^{\ *})^{1/2}$$

which are dimensionless measures of particle diameter and superficial gas velocity respectively. Note that these parameters can be varied independently by varying the actual particle diameter or superficial gas velocity. The particle Reynolds number, Re_p, is the product of d_p^* and U^*.

These parameters reflect some aspect of the flow about a particle, what can be called the micro-scale hydrodynamics. These are crucial for transition phenomena like the onset of fluidization and the onset of bubbling. There are other phenomena, like the bubble rise velocity and the variation of bubble size with height, for which the micro-scale hydrodynamics are unimportant. For an isolated bubble in a fluidized bed, there is a relationship between the rise velocity and bubble diameter

$$\frac{{u_b}^2}{gd_b} = 0.51$$

where the dimensionless group is a Froude number for the bubble. There are other Froude numbers for bubble phenomena: two for a freely bubbling bed can be found in Appendix B. Note that the microscale does not enter into these relations.

For particles with a broad distribution of sizes, there is no obvious particle diameter to use. The reciprocal mass average

$$\overline{d_p} = \sum_i \frac{x_i}{d_{pi}}$$

has theoretical justification in the creeping flow regime. It has been found (for example, Farkas [1988]) to work reasonably well for fluidization also.

The particle classification of Geldart [1973] can be considered as an identification of transition behaviour based on particle size and density. Thus the microscale dimensionless groups are used to delineate the boundaries. Grace [1986] gives an analysis of the classification; what follows is an summary of his analysis.

The group C particles are cohesive and are not normally used in FBC. The group A particles are larger, they fluidize evenly and permit uniform fluidization before bubbling occurs. This uniform state is caused by interparticle forces which stabilize the bed. As the particles get larger and more dense the importance of these forces is diminished. When the

hydrodynamic forces dominate the interparticle forces, the particles belong to group B. Beds of these particles bubble as soon as they fluidize. The boundary between A and B particles is fitted by

$$(d_p^*)_{AB} = 101\left[\frac{\Delta\rho}{\rho}\right]^{-0.425} \qquad \text{(A.1)}$$

If the material permits large electrostatic charges or there is an external force field (other than gravity) this boundary will not hold.

Group D has the largest particles and is distinguished by having flat or wakeless bubbles. The boundary is determined by a transition in the interstitial flow, from viscously dominated to inertially dominated. This is established from the terms in the correlation for U_{mb} (1.11), written as

$$Re_{mb}^{\ 2} + 2C_1 Re_{mb} - C_2 Ar = 0 \qquad \text{(A.2).}$$

The first two terms represent the inertial and viscous terms respectively; setting them equal gives

$$Ar_{B:D} = \frac{8C_1^{\ 2}}{C_2} \qquad \text{(A.3)}$$

Thus group D beds have interstitial flow with significant inertial effects from the start of fluidization. This interpretation has an interesting consequence for Group B beds at high velocities. Since interstitial velocity is increased by the presence of bubbles a transition to group D behaviour might occur with large bubble fractions. There is an alternate physical interpretation, in terms of spouting behaviour, which would not allow such an effect.

Using the values $C_1 = 25.28$ and $C_2 = 0.0571$ in (A.3) gives the boundary as

$Ar_{B:D} = 8.95 \times 10^4$.

For the fluidizing gas being air at FBC temperatures the critical particle diameter is around 3 mm. Thus most FBC units use class B particles according to this analysis.

B. Derivation of Bubble Fraction Formula

The expansion of fluidized beds can be estimated using some variation of the two-phase theory. The average volumetric flow of bubbles is

$$\dot{V}_b = \frac{1}{L_{DB}} \int_0^{L_{DB}} u_b(z)\epsilon_b(z)dz \tag{B.1}$$

Two-phase theory holds that the flow beyond that for minimum bubbling is carried in the bubbles. Let $Y < 1$ be the correction factor to account for the through flow, then

$$\dot{V}_b = Y(U - U_{mb})A_c \tag{B.2}$$

Since ϵ_b is constant over the Dense Bed region, (B.1) and (B.2) can be combined to give

$$\epsilon_b = \frac{Y(U - U_{mb})}{\frac{1}{L_{DB}} \int_0^{L_{DB}} u_b(z)dz} \tag{B.3}.$$

The bubble rise velocity is calculated from

$$u_b(z) = 0.71\sqrt{gd_b(z)} + U - U_{mb} \tag{B.4}$$

for some expression for bubble size with vertical position. One possible choice is the formula of Darton et al. [1977]

$$d_b(z) = 0.54(U - U_{mb})^{0.4}(z + 4\sqrt{A_D})^{0.8} g^{-0.2} \tag{B.5}$$

where A_D is the catchment area of a distributor orifice.

A comment on (B.3) is in order at this point. Since ϵ_b is a constant with respect to vertical position, as verified experimentally, the integral could be taken over any subinterval of the Dense Bed and should give the same value. This is not the case. A possible explanation is that Y is not a constant but an average like that for the bubble velocity. The ratio of the averages must be constant, and so would require

$$Y(z) - Y(0) \propto u_b(z) - u_b(0) \tag{B.6}.$$

but this is only a conjecture.

Rewrite (B.5) as

$$\frac{gd_b(z)}{(U - U_{mb})^2} = 0.54\left[\frac{(U - U_{mb})^2}{g(z + 4\sqrt{A_D})} \right]^{-0.8} \tag{B.7}$$

which shows it to be a relationship between two bubble Froude numbers, or a dimensionless bubble diameter correlated to a dimensionless vertical position. The denominator of (B.3) is approximated by using a midpoint rule for the integral, giving

$$\epsilon_b = \frac{Y(U - U_{mb})}{u_b(0.5L_{DB})} = \frac{Y}{0.52Fr_{DB}^{-0.4} + 1} \tag{B.8}$$

where

$$Fr_{DB} = \frac{(U - U_{mb})^2}{g(0.5L_{DB} + 4\sqrt{A_D})} \quad .$$

Johnsson et al. [1989] used $Y = 0.3$ to fit their measured bubble fractions;[10] this derivation does not differ substantially from theirs, though the final form is different. Note that Y is the asymptotic value for ϵ_b as Fr_{DB} gets large. In that case the value of 0.3 seems low; perhaps Y is a weak function of the excess gas velocity.[11]

---

10 This value is also used by Glicksman et al. [1987], see their equation (15).

11 In light of the recent and on-going work on bed expansion being conducted at Chalmers University much of this work will have to be revised.

C. Adjustment of Interstitial Flow due to Bubbles

Consideration of the bubble model of Davidson and Harrison [1963] shows that the interstitial gas flow is increased in the neighbourhood of a bubble. Furthermore, the calculations of gas flow between a pair of bubbles shows an extension of the region of increased interstitial gas flow, and an extension of the bubble cloud (Clift et al. [1972]). Hence, for a freely bubbling bed one should expect the interstitial flow to increase above the minimum bubbling value, and this increase should be related to the bubble volume fraction.

Clift et al. [1984] present such an adjustment and claim this is supported by aerosol capture measurements. These measurements were performed on a 0.15 m bed of class B particles, for superficial velocities from U_{mb} to $2.5U_{mb}$. Their relation is

$$U_e = U_{mb}(1 + 1.5\,\epsilon_b^{2/3}) \tag{C.1}$$

but their derivation is unsatisfactory in two places. Below a revised derivation is presented and the result is shown to be very close to (C.1) in the range of experimental measurements.

Consider a spherical bubble of diameter d_b in a fluidized bed with bubble fraction ϵ_b. Associate with this bubble a spherical shell of diameter

$$\frac{d_b}{\epsilon_b^{1/3}}$$

centred on the bubble. Since

$$\sum_{\text{bubbles}}^{\text{all}} \frac{\pi}{6} d_b^{\,3} = \epsilon V_{DB}$$

the shells will enclose the full bed volume, regardless of the sizes or distribution of sizes of the bubbles. The surface area of the cell's upper hemisphere is

$$\frac{\pi d_b^{\,2}}{2\epsilon_b^{2/3}} .$$

From the Davidson - Harrison model, the gas flows through the bubble at volumetric rate

$$\frac{3\pi d_b^{\,2} U_{mb}}{4} .$$

Without the bubble, the flow would be

$$\frac{\pi d_b^{\,2} U_{mb}}{4}$$

so the excess is

$$\frac{\pi d_b^{\,2} U_{mb}}{2}$$

However, the interstitial velocity is no longer U_{mb} but U_e, due to the presence of bubbles. For sufficiently close bubbles, or for open ('slow') bubbles, all this excess will flow out the upper hemisphere of the shell. This gives an average excess velocity of

$$U_e\,\epsilon_b^{2/3}$$

so that

$$U_e = U_{mb} + U_e \epsilon_b^{2/3}$$

or

$$U_e = \frac{U_{mb}}{1 - \epsilon_b^{2/3}} \qquad \text{(C.2).}$$

The singularity at $\epsilon_b = 1$ is to be expected because then there is no longer any emulsion. This singularity is not present in (C.1). A more sophisticated construction of shells could be made that would move the singularity to some $\epsilon_b < 1$, representing maximal packing of the bubbles, but some assumption of bubble size distribution would be needed.

Since (C.1) is known to work for the measurements in Clift et al. [1984] then (C.2) cannot differ too much, at least in the lower range of ϵ_b . Consider the difference ($x = \epsilon_b^{2/3}$)

$$E(x) = 1 + 1.5x - \frac{1}{1-x} = \frac{0.5x\,(1-3x)}{1-x} \qquad \text{(C.3).}$$

This has an extremum at $\dot{x} = 1 - \sqrt{2/3} \simeq 0.184$ ($\dot{\epsilon}_b \simeq 0.08$) where the difference is

$E(\dot{x}) \simeq 0.051$.

With a maximum difference around 5%, (C.2) must give results comparable to those of (C.1), and so must fit the experimental data equally well. The range of superficial velocity is below that normally used for FBC, and so (C.2) needs to be tested at higher velocities.

D. Bubble Growth Formulae

The average bubble size in a fluidized bed is a function of the excess gas velocity, column diameter, distributor design and vertical position. It is not clear if there is a radial size distribution. For small-particle beds non-uniformity has been found, see, for example, Werther [1974]. Glicksman et al. [1987] observed uniform cross-sectional distribution in their large-particle bed. Uniformity will be assumed, if only because the non-uniformity cannot be predicted.

There is a multiplicity of formulae that claim to predict the variation of mean bubble diameter with vertical position. Two of long-standing and wide acceptance are due to Mori and Wen [1975] and Darton et al. [1978]. Each was derived and tested for small-particle beds. More recent is the formula of Glicksman et al. [1987] which was derived for large-particle beds, large bed cross-section, and high superficial velocity. That paper also has some experimental measurements of bubble size at different heights and velocities. The data will be used to choose among the three formulae.

The formula of Mori and Wen is

$$d_b = d_{b\infty} - (d_{b\infty} - d_{bo})e^{-0.3z/d_c} \qquad (D.1)$$

where 0.3 in the exponent is a fitted parameter. The maximum bubble size, $d_{b\infty}$, is

$$d_{b\infty} = 2.35 d_c \left[\frac{(U - U_{mb})^2}{g d_c} \right]^{0.2} \qquad (D.2)$$

The initial bubble size for a porous plate distributor is

$$d_{bo} = 3.69 \frac{(U - U_{mb})^2}{g} \qquad (D.3)$$

while for a multi-orifice distributor

$$d_{bo} = 1.38 \sqrt{A_D} \left[\frac{(U - U_{mb})^2}{g\sqrt{A_D}} \right]^{0.2} \qquad (D.4)$$

Miwa et al. [1972] showed that when (D.4) predicts overlapping bubbles the distributor is behaving like a porous plate and (D.3) can be used. This was confirmed for large-particle beds by Glicksman et al. [1987].

The formula of Darton is

$$d_b = 0.54 z^* \left[\frac{(U - U_{mb})^2}{g z^*} \right]^{0.2}, \quad z^* = z + 4\sqrt{A_D} \qquad (D.5)$$

Here 0.54 is the fitted parameter. Note that the initial bubble size predicted by (D.5) differs from (D.4) only by a multiplicative constant, 1.64 in place of 1.38. This is because the derivation of (D.5) uses similar mechanisms as used in the derivation of (D.4). The difference comes from the fit parameter. When the initial bubbles overlap, the catchment area A_D must be changed to give (D.3); the change is

$$A_D = 7.63 \frac{(U - U_{mb})^4}{g^2} \qquad (D.6)$$

Both these formulae give bubble size by direct calculation.

Glicksman et al. [1987] give an implicit formula for bubble diameter

$$\psi(d_b) - \psi(d_{bo}) = 0.722 \frac{g(z - z_o)}{(U - U_{mb})^2} \qquad (D.7)$$

where

$$\psi(d_b) = (Fr_b + 1)^{9/4}(Fr_b - 4/9) \qquad (D.8)$$

$$Fr_b = \frac{0.71\sqrt{gd_b}}{(U - U_{mb})} \qquad (D.9)$$

This equation must be solved numerically. Both the multiplier (0.722) and the exponent (9/4) are parameters fitted in the course of the derivation of (D.7). The initial values are given as for the formula of Mori and Wen, with $z_o = 0.5d_{bo}$.

Figures D.1 to D.3 show the predictions of the three formulae with data taken from Figure 14 of Glicksman et al. [1987]. Clearly the formula of Mori and Wen is the best. The Glicksman's formula is the worst of the three and requires the most computation. It is puzzling that Glicksman's formula does not fit the data very well. This is possibly because the free parameters are fit to data not directly related to bubble size, namely the deviation from two-phase theory and the bubble passage frequency. Either there are invalid assumptions between the fitting and the formula (D.7), or the model is sensitive to the errors in those fit parameters. It is also worth noting that Weluda et al. [1987] found that the formula of Mori and Wen worked well for their measured bubble sizes in a hot (850°C) combustor, using particles around 0.3 mm.

E. Fluidization Dynamics and Modelling of AFBC[12]

This paper contains an outline of an alternative model for the hydrodynamics and gas-solid contacting under AFBC operating conditions. While the model is being developed for the Queen's University Pilot Plant, we expect it to be generally applicable with few modifications.

First, AFBC operating conditions are examined and shown to give very large initial bubble diameters. This is followed by an analysis of the hydrodynamics of a shallow slugging bed. The implications for gas-solid reactions are investigated and the resulting combustion model is outlined.

AFBC Operating Conditions

AFB combustors normally operate with a large mean particle size, to avoid entrainment losses. Thus the bed requires a high gas velocity to attain fluidization, Table E.1. To promote good mixing of particles, the superficial velocity should be greater than twice the minimum bubbling velocity. Alternatively, if the superficial velocity is set, for example to give a desired excess air level, there is an upper bound on the mean particle size in the bed, Table E.2.

Consider now the size of bubbles formed at the distributor under these conditions. For an orifice, the Davidson and Schuler expression is

$$d_{bo} = 1.38(U - U_{mb})^{0.4} A_D{}^{0.4} g^{-0.2} \qquad (E.1)$$

while for a continuous distributor, Mori and Wen give

$$d_{bo} = 3.69 \frac{(U - U_{mb})^2}{g} . \qquad (E.2)$$

For sufficiently high superficial velocities, the bubbles predicted by (E.1) overlap, in which case the distributor may be considered as continuous and (E.2) used to predict initial bubble size. This procedure was recently verified by Glicksman et al. [1987] with 1 mm. particles. Table E.3 contains the predicted bubble sizes over the AFBC range of excess gas velocity.

The Queen's University Pilot Plant has a 38 cm x 40 cm cross-section and operates with extrapolated bed heights (based on mean pressure distribution along the wall) of 50 to 80 cm. The separation of nozzles in the distributor plate is less than 4 cm. Normal operation has a superficial velocity between 1 and 3 m/s and bed particle size between 0.5 and 1 mm. Thus the distributor is effictively continuous and the initial bubble size is comparable to, or larger than the bed. The bed is commonly slugging and the bed height allows only a single slug. Industrial units also have initial bubble sizes comparable to the bed height so we expect similar behaviour.

Hydrodynamics

The following is the simplest analysis of the cyclic, single slug phenomena as shown in Figure E.1.

Assuming a well-defined bed height L, a periodic function of time, a mass balance of gas in the bed gives

$$\frac{dL}{dt} + U_{mb} = U \qquad (E.3)$$

---

12 This is the paper, Becker and Gogolek [1988], presented at the IEA Mathematical Modelling Meeting in Palo Alto in May, 1988.

since the solid particles at the top of the bed are at minimum bubbling conditions. At the start of a cycle the bed is at L_{mb}, so integrating (E.3) from this state gives

$$L - L_{mb} = (U - U_{mb})t \qquad (E.4)$$

for $0 \le t \le \tau$, the period of the cycle. Let $f = \frac{1}{\tau}$ be the frequency of the cycle, i.e. the slug bursting frequency, then the maximum bed height is

$$L_{\max} = L_{mb} + \frac{(U - U_{mb})}{f} . \qquad (E.5)$$

Experimental correlations of the dominant frequency of pressure fluctuations in slugging beds are readily available. Broadhurst and Becker [1976] report

$$f = 0.34 \sqrt{\frac{g}{d_c}} \left[\frac{d_c}{L_{mb}} \right]^{0.85} \qquad (E.6)$$

where d_c is the column diameter, for particles smaller than 0.5 mm. Noordergraaf et al. [1987] found

$$f = 0.32 \sqrt{\frac{g}{d_c}} \qquad (E.7)$$

for particles from 0.45 mm to 3 mm. There is good agreement where the range of aspect ratios overlap, $\frac{1}{2} < \frac{L_{mb}}{d_c} < 3$. Using (E.7) in (E.5) gives

$$\frac{L_{\max} - L_{mb}}{d_c} = 3.13 \frac{U - U_{mb}}{\sqrt{g d_c}} \qquad (E.8)$$

and this is supported by the experimental results of Farkas [1988].

The volume of the slug, as a function of time, is

$$V_{slug} = A_c (L - L_{mb}) = A_c (U - U_{mb})t \qquad (E.9)$$

where A_c is the cross-sectional area of the column. At bursting, the slug volume is

$$V_{slug\ \max} = A_c (L_{\max} - L_{mb}) = 3.13 A_c d_c \frac{U - U_{mb}}{\sqrt{g d_c}} \qquad (E.10)$$

Reaction Modelling: Gas-Solid Contacting

A continual rain of particles falls through the developing slug. After the slug bursts, there is a much heavier shower of particles as the bed returns to the original state. These observations lead to the following model.

The gas is transported in plug flow in the slug, through the descending rain of particles. Since all the bed mass must somehow fall, the average downward particle mass flux is

$$\dot{m}_d = \frac{m_b}{\tau} = m_b f = (1 - \epsilon_{mb}) \rho_p A_c L_{mb} f \qquad (E.11)$$

of which a fraction β_d rains through the slug. Assuming the particle velocity through the slug is similar to the free fall velocity,

$$v_d = c_1 \sqrt{g(L_{max} - L_{mb})}\,, \tag{E.12}$$

and using (E.8), we obtain

$$v_d = 1.77 c_1 \sqrt{(U - U_{mb})\sqrt{g d_c}}\,. \tag{E.13}$$

The concentration of particles dispersed in the slug is

$$C_d = \frac{\beta_d \dot{m}_d}{A_c v_d} \tag{E.14}$$

and by (E.7), (E.11) and (E.13)

$$C_d = \frac{0.18}{c_1} \beta_d (1 - \epsilon_{mb}) \rho_p \frac{L_{mb}}{d_c} \left[\frac{\sqrt{g d_c}}{U - U_{mb}} \right]^{1/2} \tag{E.15}$$

The mean residence time of particles in the slug is

$$\theta_d = \frac{L_{max} - L_{mb}}{v_d} \tag{E.16}$$

or

$$\theta_d = \frac{1.77}{c_1} \left[\frac{U - U_{mb}}{\sqrt{g d_c}} \frac{d_c}{g} \right]^{1/2} \tag{E.17}$$

and the mean residence time of the gas in the slug is

$$\theta_g = \tau = 3.13 \sqrt{\frac{d_c}{g}} \tag{E.18}$$

A schematic of the combustor model is given in Figure E.2. The dense phase (Compartment A) has no gas and has particle mass

$$m_b \left[1 - \beta_d \left(\frac{U - U_{mb}}{v_d} \right) t f \right].$$

The dilute phase (Compartment B) has the gas in plug flow and falling dispersed particles of total mass

$$m_b \beta_d \left(\frac{U - U_{mb}}{v_d} \right) t f.$$

These premises have the following implications for combustion reactions. The dense phase only allows pyrolysis while combustion only occurs in the dilute phase and the freeboard. The dilute phase provides modest particle residence times but relatively high char and oxygen concentrations for char combustion. The freeboard may provide greater residence time but the char and oxygen concentrations are lower.

Summary

In this model bubble assemblies are abandoned as unrealistic for AFBC conditions. The hydrodynamics are given by a cyclic, single slug model. In this model, gas is transported in plug flow and encounters a steady rain of particles. Combustion reactions occur only in this dilute phase or in the freeboard.

Details remain to be worked out before the model can be used for calculations and compared with experimental data.

d_p mm	$Ar^{1/3}$	Re_{mb}	U_{mb} m/s
0.5	8	0.39	0.11
1	16	2.5	0.36
2	32	15	1.07
2.6	41	27	1.5
5	80	110	3.0

Table E.1. Predicted Minimum Bubbling Velocity of Sand Using Air @ 850°C

U m/s	$d_{p\ upper}$ mm
1	1.2
2	1.9
3	2.6

Table E.2. Upper Bound on Particle Size

$U - U_{mb}$ m/s	d_{bo} cm
0.5	9.4
0.7	18
1.0	38
1.5	85
2.0	151
3.0	339

Table E.3. Initial Bubble Diameters for Continuous Distributors

References

S. Andersson [1988]
Discussion on Mathematical Modelling of Circulating Fluidized Bed Boilers, presented at the IEA-AFBC Mathematical Modelling Meeting, May, 1988, Palo Alto, California

H.A. Becker, P.E.G. Gogolek [1988]
Fluidization Dynamics and Modelling of AFBC , presented at the IEA-AFBC Mathematical Modelling Meeting, May, 1988, Palo Alto, California

G. Birkhoff [1964]
Averaged Conservation Laws in Pipes, J. Math. Anal. Appl., v. 8, pp. 66 - 77

C. Brereton [1987]
Fluid Mechanics of High Velocity Fluidized Beds, Ph.D. Thesis, University of British Columbia

T. Broadhurst, H.A. Becker [1973]
The Application of the Theory of Dimensions to Fluidized Beds, in Fluidization and Its Applications, eds. H. Angelino, J.P. Couderc, H. Gibert, G. Laguerie

T.E. Broadhurst, H.A. Becker [1976]
Measurement and Spectral Analysis of Pressure Fluctuations in Slugging Beds in Fluidization Technology Vol. I, ed. D.L. Keairns, pp. 63 - 85

G.S. Canada, M.H. McLaughlin, F.W. Staub [1978]
Flow Regimes and Void Fraction Distribution in Gas Fluidization of Large Particles in Beds Without Tube Banks, A.I.Ch.E. Symp. Ser. No. 176, v. 74, pp. 14 - 26

R. Clift, M. Ghadiri, J.L. Monteiro, B.K.C. Tan, K.V. Thambimuthu [1984]
Interphase Transfer and Interstitial Flow in Bubbling Bed, Inferred from Measurements of Aerosol Capture, Fluidization IV, D. Kunii, R. Toei (eds.), pp. 85 - 92

R. Clift, J.R. Grace, J.R. Cheung, T.H. Do [1972]
Gas and solids motion around deformed and interacting bubbles in fluidized beds, Journal of Fluid Mechanics, v. 51, pp. 187 - 205

R.R. Cranfield, D. Geldart [1974]
Large Particle Fluidization, Chem. Eng. Sci., v.29, pp. 935 - 947

R.C. Darton, R.D. LaNauze, J.F. Davidson, D. Harrison [1977]
Bubble Growth due to Coalescence in Fluidised Beds, Trans. Inst. Chem. Eng., v.55, pp. 274 - 280

J.F. Davidson, D. Harrison [1963]
Fluidised Particles, Cambridge University Press

N.A. Farkas [1988]
Fluidization Properties of Silica Sand and of Bed Material from Fluidized Bed Combustion, M.Sc. Thesis, Queen's University at Kingston

T.J. Fitzgerald [1985]
Coarse Particle Systems, in Fluidization, J.F. Davidson, R. Clift, D. Harrison (eds.) pp. 413 - 435

D. Geldart [1985]
Elutriation, in Fluidization, J.F. Davidson, R. Clift, D. Harrison (eds.) pp. 383 - 412

D. Geldart [1973]
Types of Gas Fluidization, Powder Technology, v. 7, pp. 285 - 291

D. Geldart, M.J. Rhodes [1986]
From Minimum Fluidization to Pneumatic Transport - A Critical Review of the Hydrodynamics, Proc. 1st Int. Conf. on CFB, pp. 21 - 31

L.R. Glicksman, W.K. Lord, M. Sakagami [1987]
Bubble Properties in Large-Particle Fluidized Beds, Chem. Eng. Sci., v. 42, pp. 479 - 491

P.E.G. Gogolek [1988]
Two Problems in Modelling FBC, presented at the IEA-AFBC Mathematical Modelling Meeting, November, 1988, Amsterdam

H. Görmar, U. Renz, N. Verweyen [1989]
Heat Transfer to the Cooled Freeboard of a Fluidized Bed, Proc. 1989 Int, Conf. on FBC, pp. 1241 - 1244

J.R. Grace [1986]
Contacting Modes and Behaviour Classification of Gas-Solid and Other Two-Phase Suspensions, Can. J. Chem. Eng., v. 64, pp. 353 - 363

J.R. Grace, R. Clift [1974]
On the Two-Phase Theory of Fluidization, Chem. Eng. Sci., v. 29, pp. 327 - 334

M. Horio [1988]
On the Scaling Law of Bubbling and Circulating Fluidized Beds, presented at the IEA-AFBC Mathematical Modelling Meeting, May, 1988, Palo Alto, California

M. Horio, A. Nonaka, Y. Sawa, I. Muchi [1986]
A New Similarity Rule for Fluidized Bed Scale-up, A.I.Ch.E. J. v. 32, pp. 1466 - 1482

F. Johnsson, B.-Å. Andersson, B. Leckner [1989]
Pressure Distribution and Particle Concentration in Bubbling Fluidized Beds, Fluidization VI Proceedings

B. Leckner, F. Johnsson, S. Andersson [1985]
Erosion in Fluidized Beds - Influence of Bubbles, presented at the IEA-AFBC Workshop on Materials, 1985, Nova Scotia, Canada

K. Miwa, S. Mori, T. Kato, I. Muchi [1972]
Behaviour of Bubbles in Gaseous Fluidized Beds, Int. Chem. Eng., v.12, pp. 181 - 194

S. Mori, C.Y. Wen [1975]
Estimation of Bubble Diameter in Gaseous Fluidized Beds, AIChE J., v.21, pp. 109 - 115

I.W. Noordergraaf, A. van Dijk, C.M. van den Bleek [1987]
Fluidization and Slugging in Large-Particle Systems Powder Technology, v.52 pp. 59 - 68

S.P. Sit, J.R. Grace [1981]
Effect of Bubble Interaction on Interphase Mass Transfer in Gas Fluidized Beds, Chem. Eng. Sci., v.36, pp. 327 - 335

O. Sitnai, S.C. Dent, A.B. Whitehead [1982]
Asymptotic Bubble Size in Tubed Fluidized Beds, Chem. Eng. Sci., v. 37, pp. 1429 - 1430

F.W. Staub [1979]
Solids Circulation in Turbulent Fluidized Beds and Heat Transfer to Immersed Tube Banks, J. Heat Transfer, v. 101, pp. 391 - 396

W.J. Thiel, O.E. Potter [1977]
Slugging in Fluidized Beds, Ind. Eng. Chem. Fundam., v. 16, pp. 242 - 247

K.D. Weluda, K. Schütte, W. Wittler, R. Wingelsdorf, K. Schügerl [1987]
Investigations of the Bubble Phase Behaviour in a High Temperature Fluidized Bed Reactor with Coal Combustion, Chem. Eng. Comm., v. 62, pp. 161 - 185

J. Werther [1974]
Influence of the bed diameter on the hydrodynamics of gas fluidized beds, AIChE Symp. Ser., v.70, No. 141, pp. 53 - 62

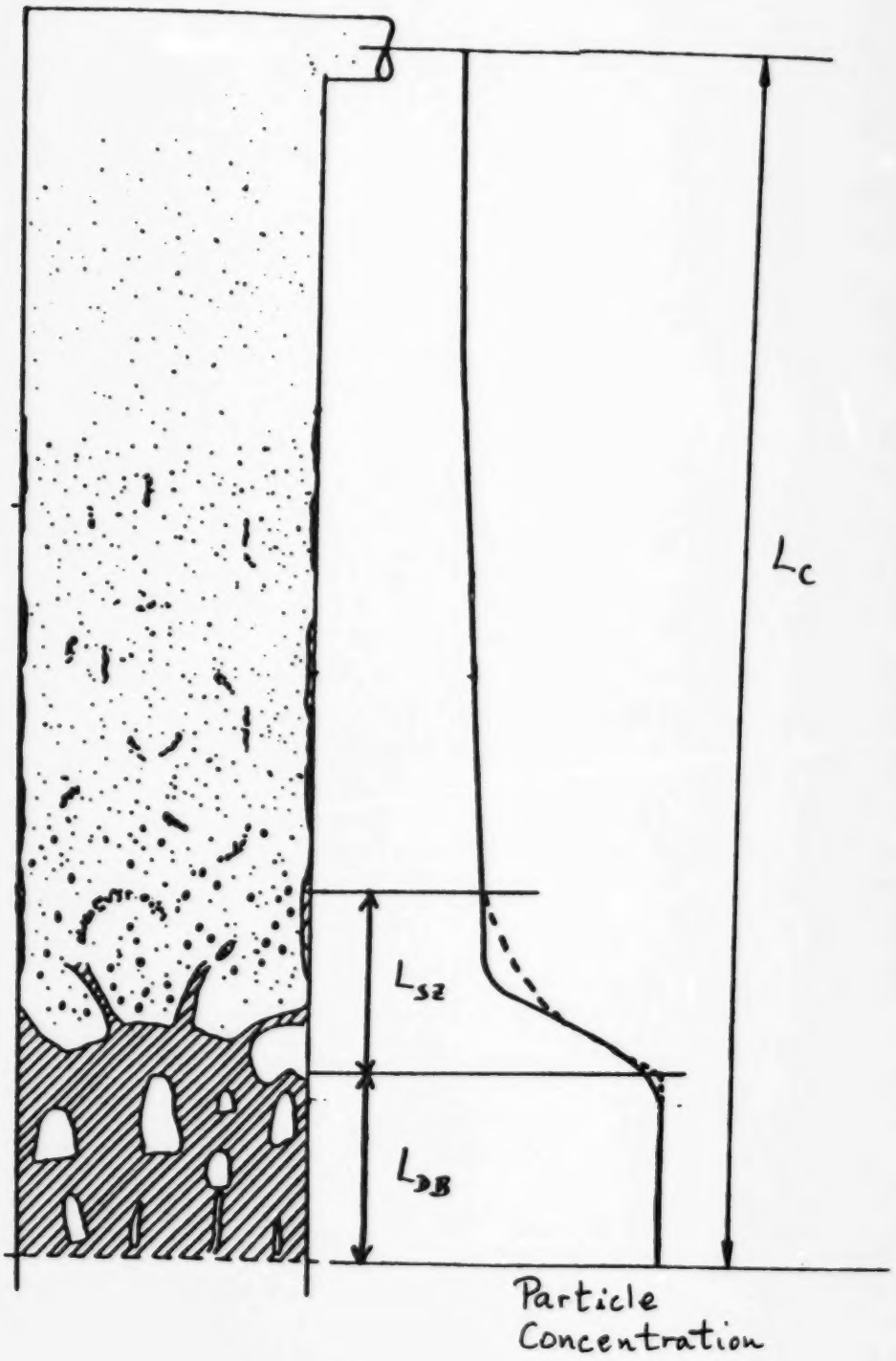


Fig. 1.1. The three zones
(after Geldart [1985])

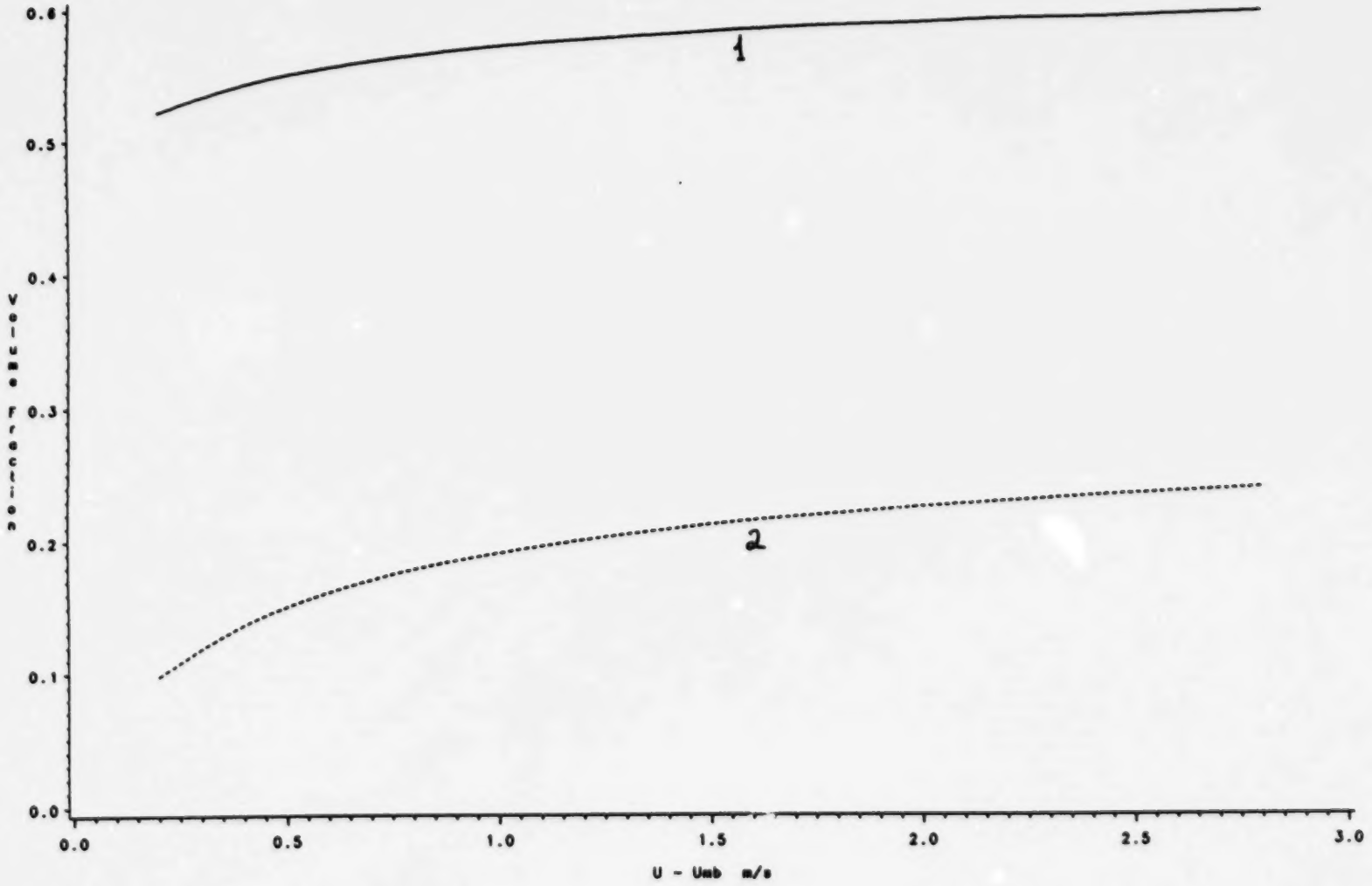


Fig. 1.2. Dense Bed expansion results, free bed

1 - bed voidage, 2 - bubble fraction

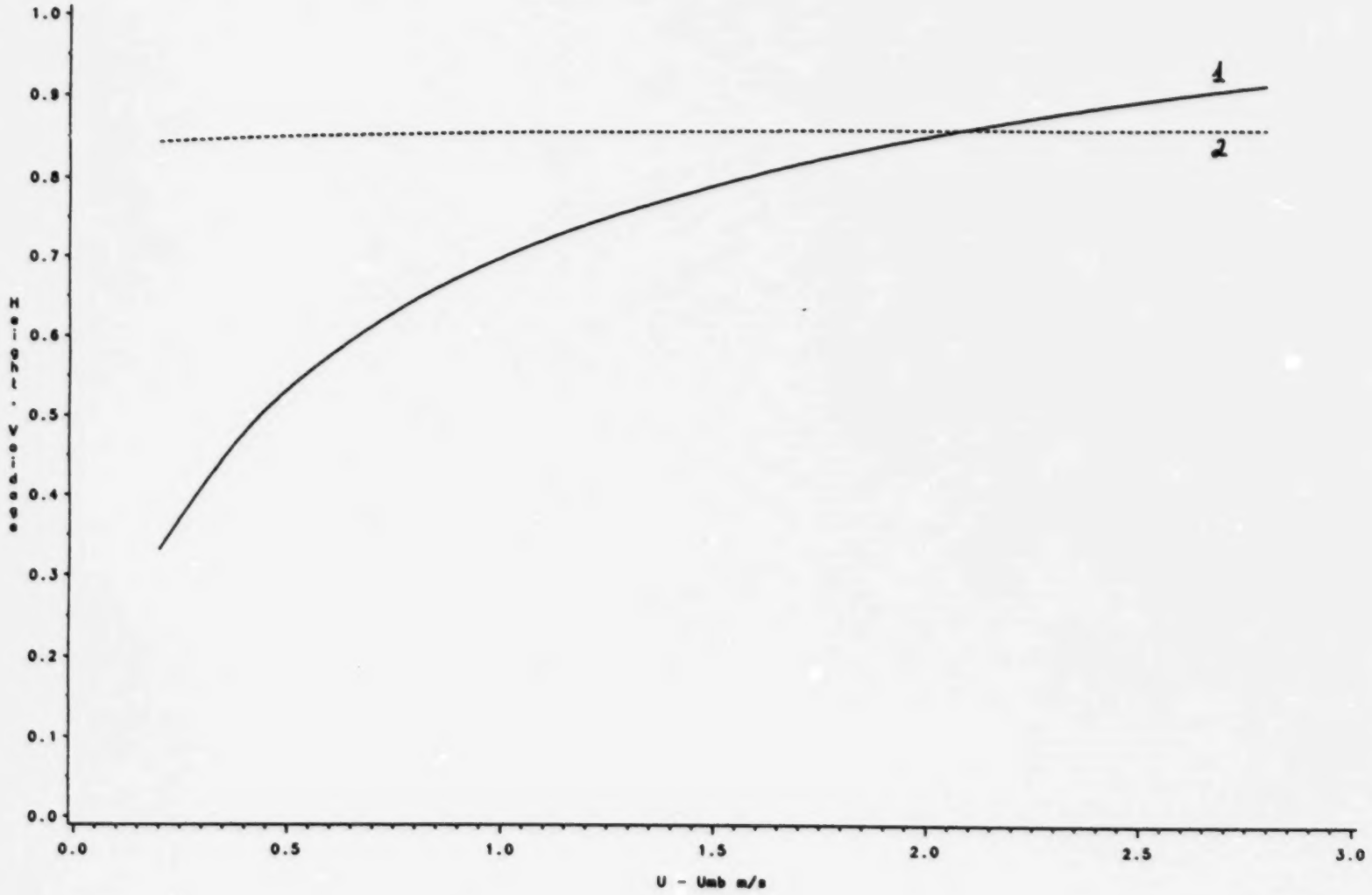


Fig. 1.3. Splash Zone expansion results, free bed

1 - Lsz/Lmb, 2 - SZ voidage

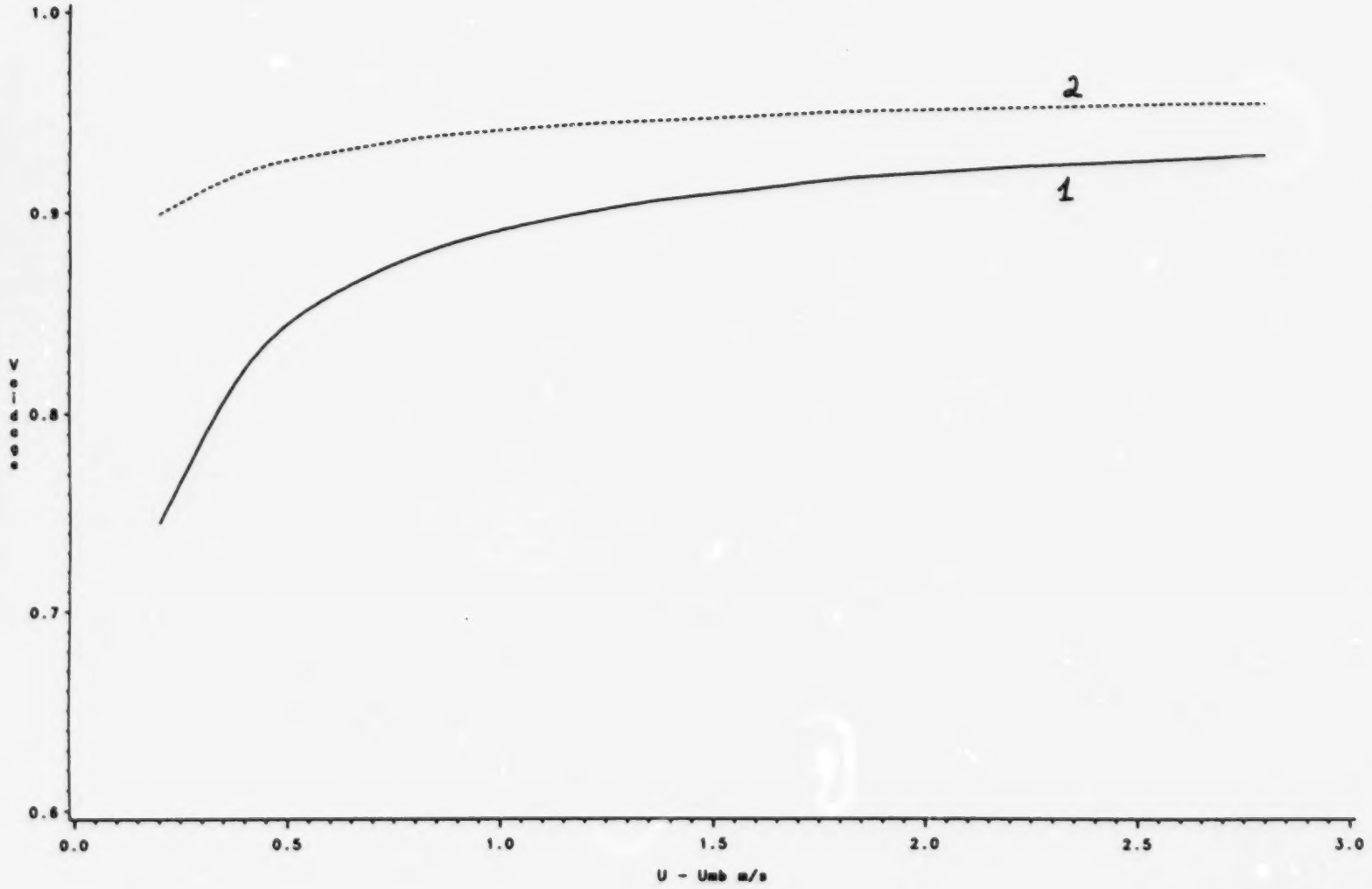


Fig. 1.4. Voidages for constrained bed

1 - Dense Bed, 2 - Splash Zone

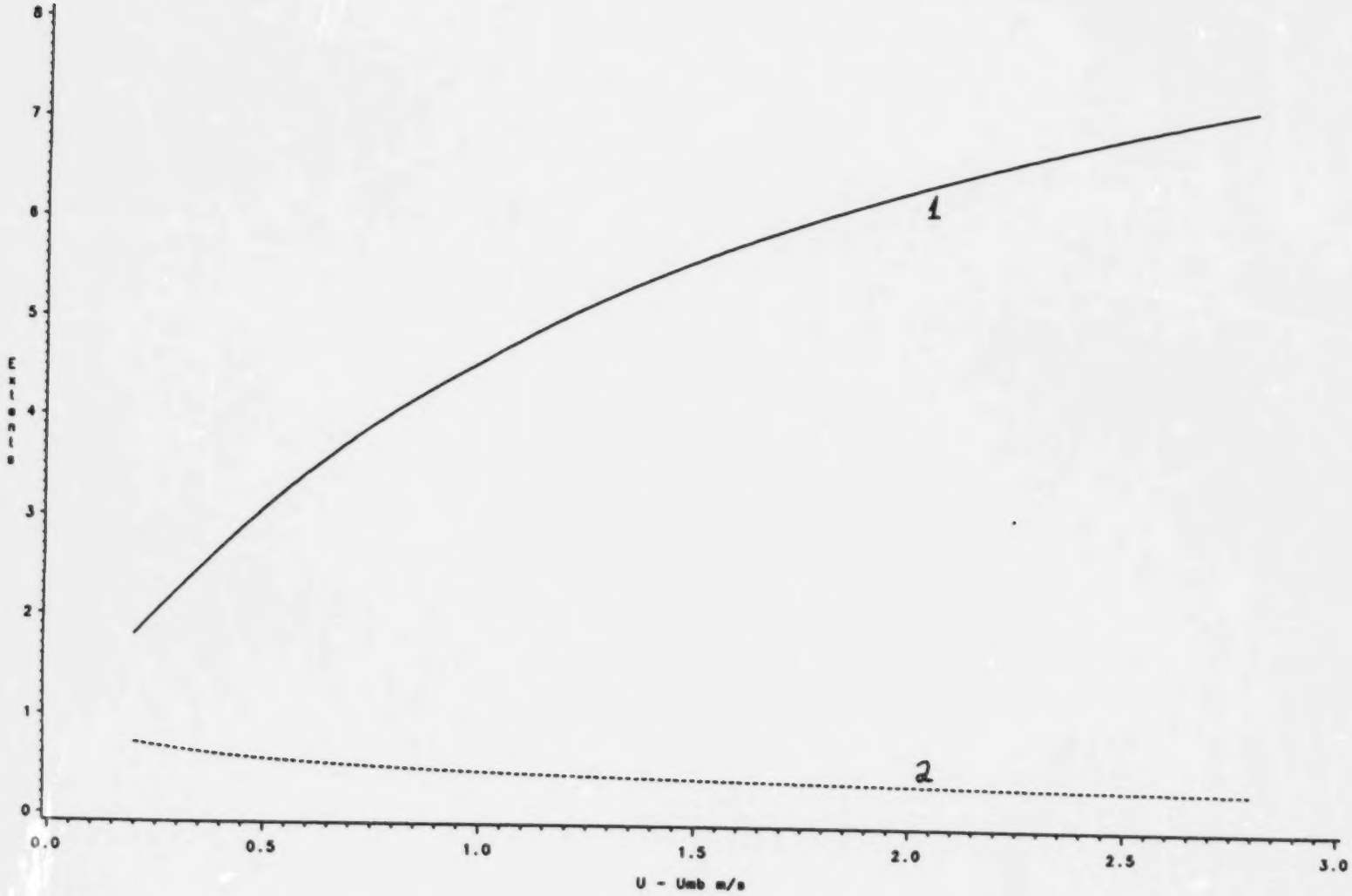


Fig. 1.5. Zone extents for constrained bed

1 - Dense Bed, 2 - Splash Zone

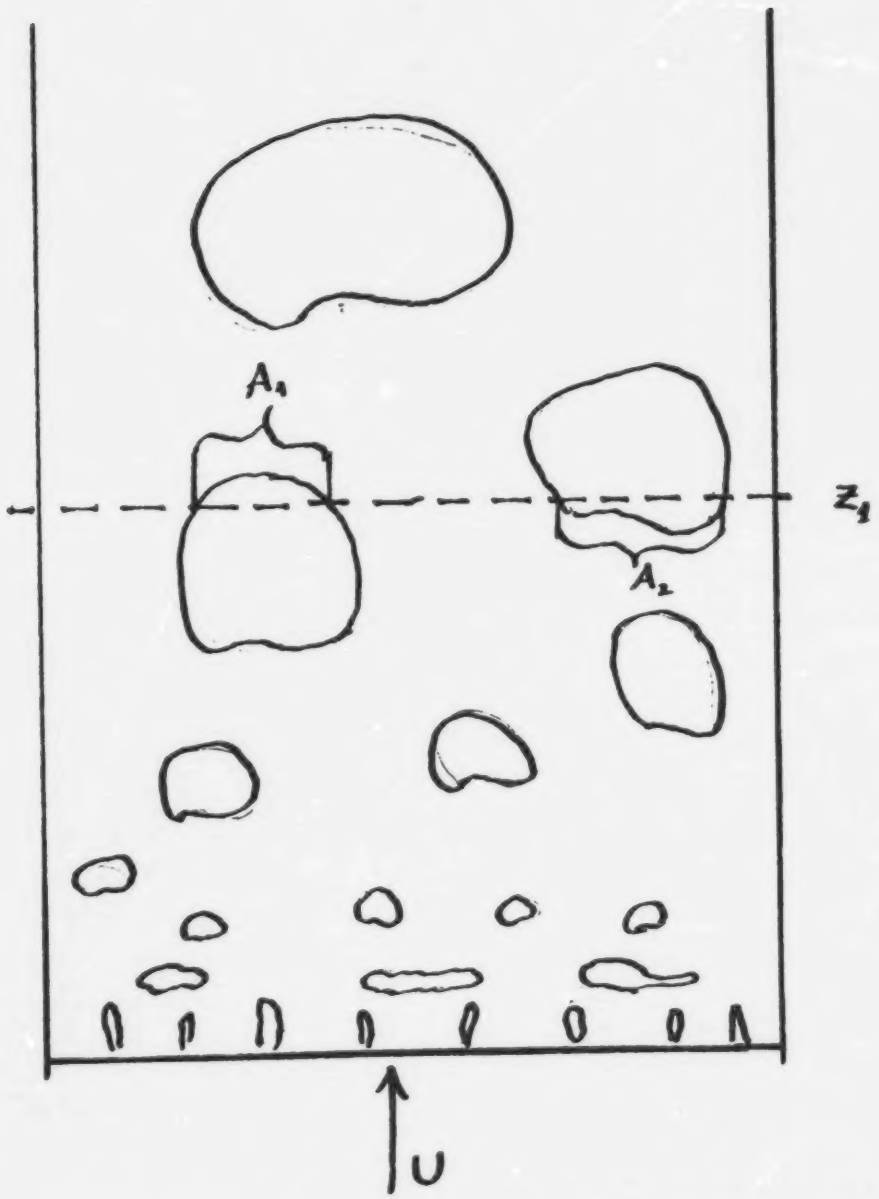


Fig. 2.1. Bubble flow schematic

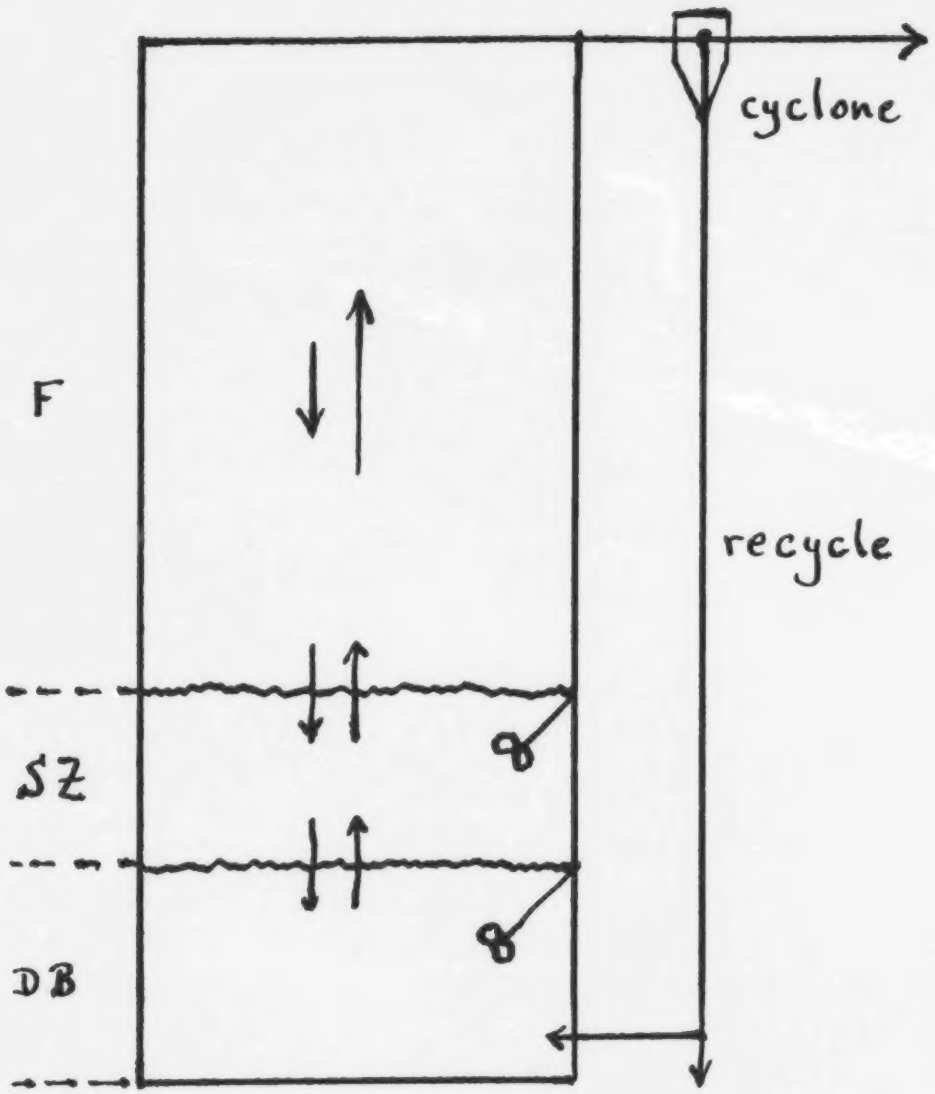


Fig. 3.1. Particle flow schematic

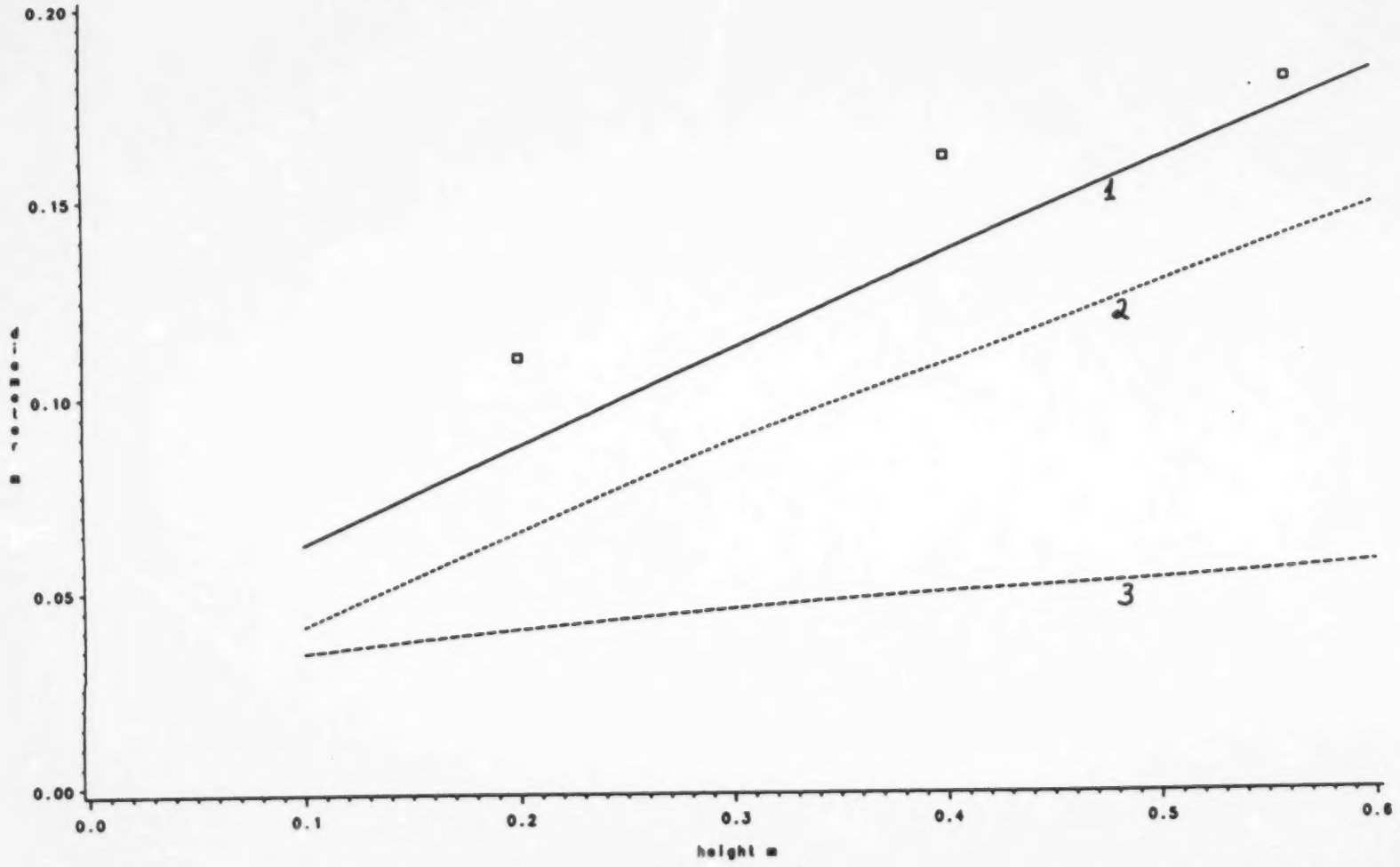


Fig. D1. Comparison of bubble size formulae

U - Umb = 0.31 m/s
1 - Mori & Wen, 2 - Darton, 3 - Clicksman
Experimental points from Clicksman

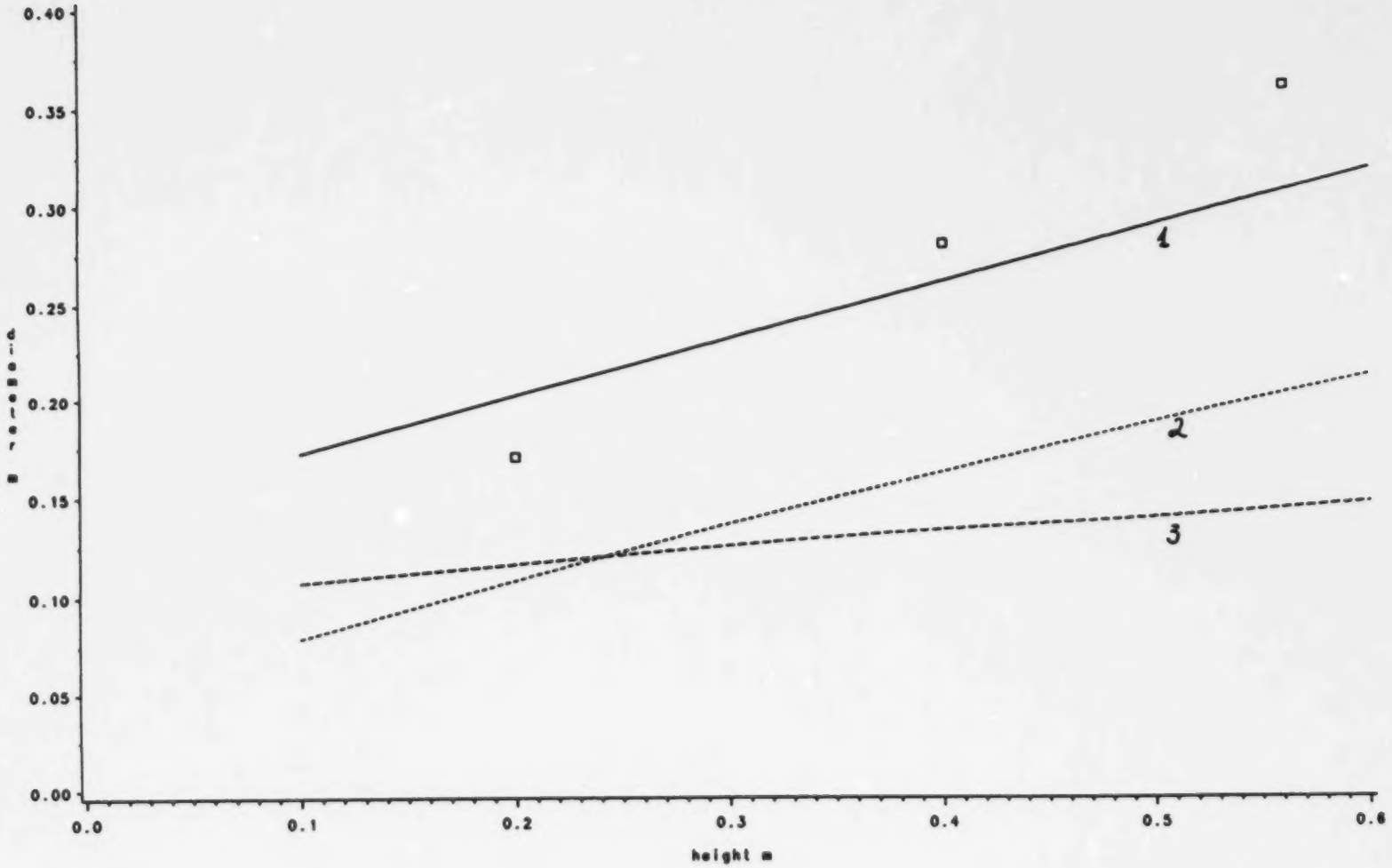


Fig. D2. Comparison of bubble size formulae

U - Umb = 0.61 m/s
1 - Mori & Wen, 2 - Darton, 3 - Glicksman
Experimental points from Glicksman

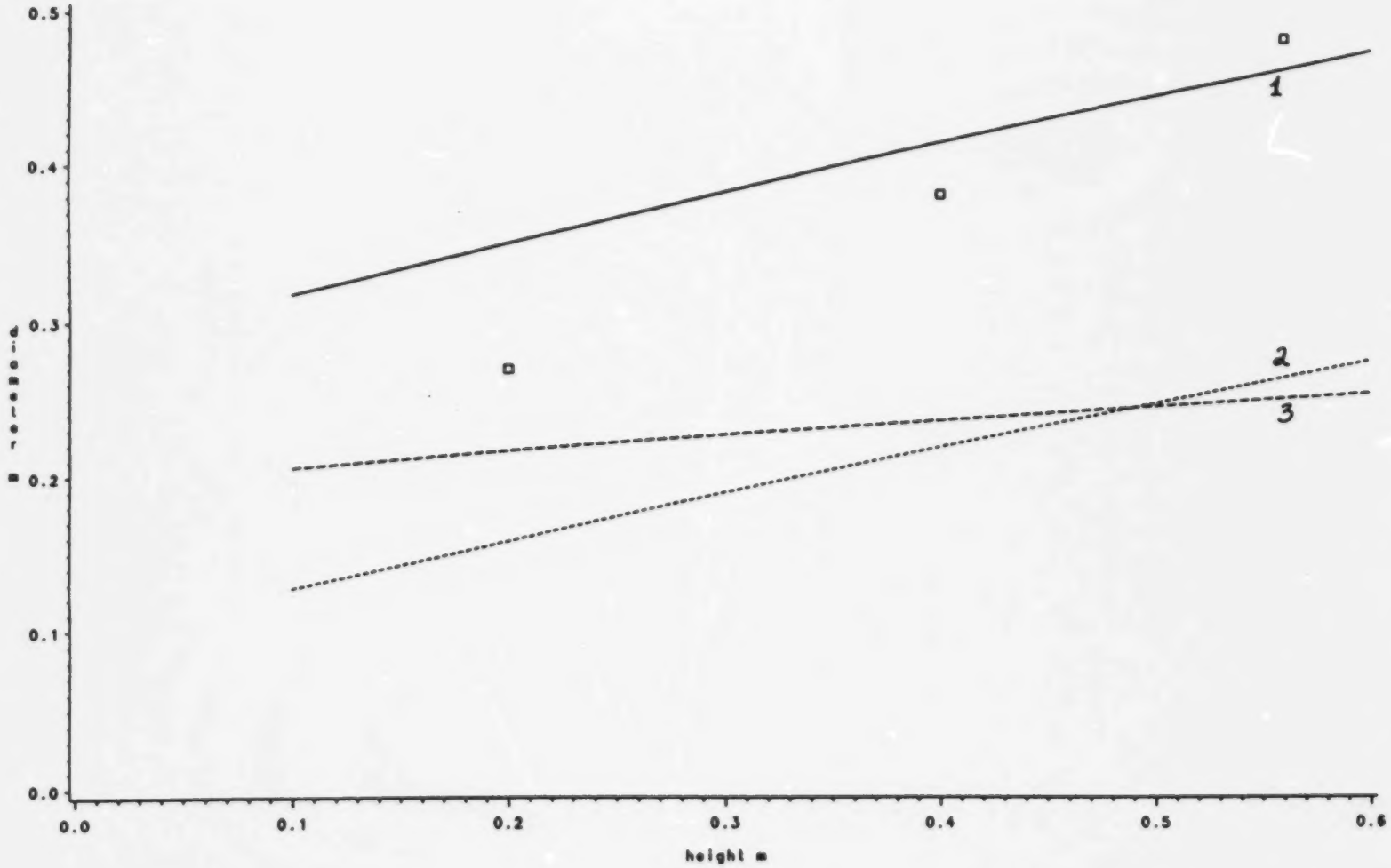
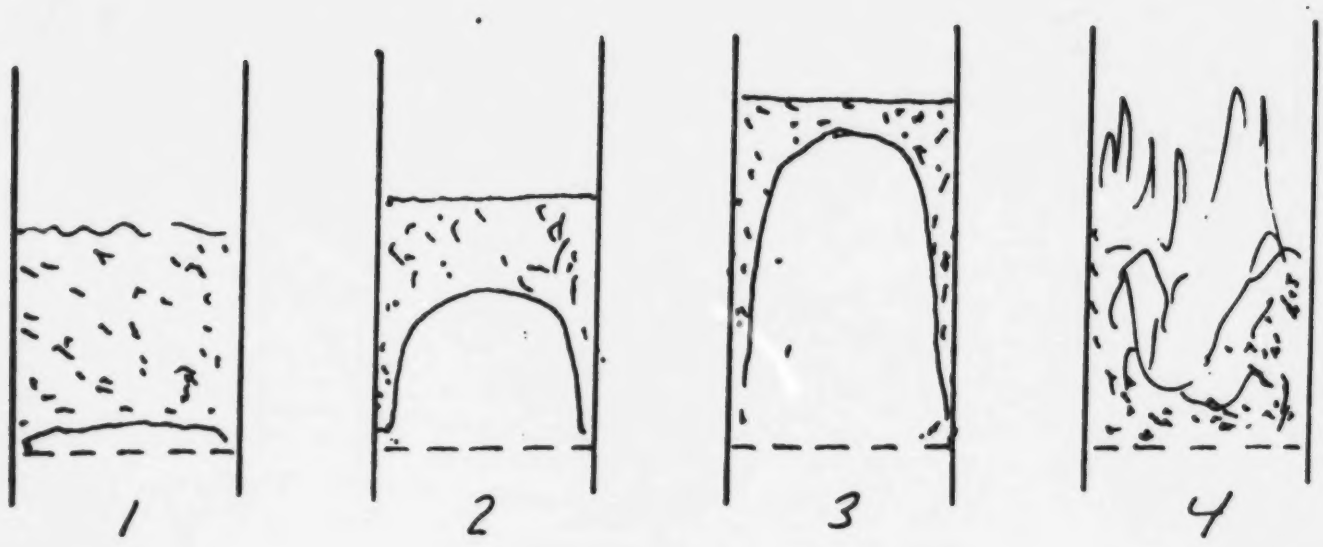


Fig. D3. Comparison of bubble size formulae

U - Umb = 0.87 m/s
1 - Mori & Wen, 2 - Darton, 3 - Glicksman
Experimental points from Glicksman



cycle repeats

Fig. E.1. Single slug behaviour

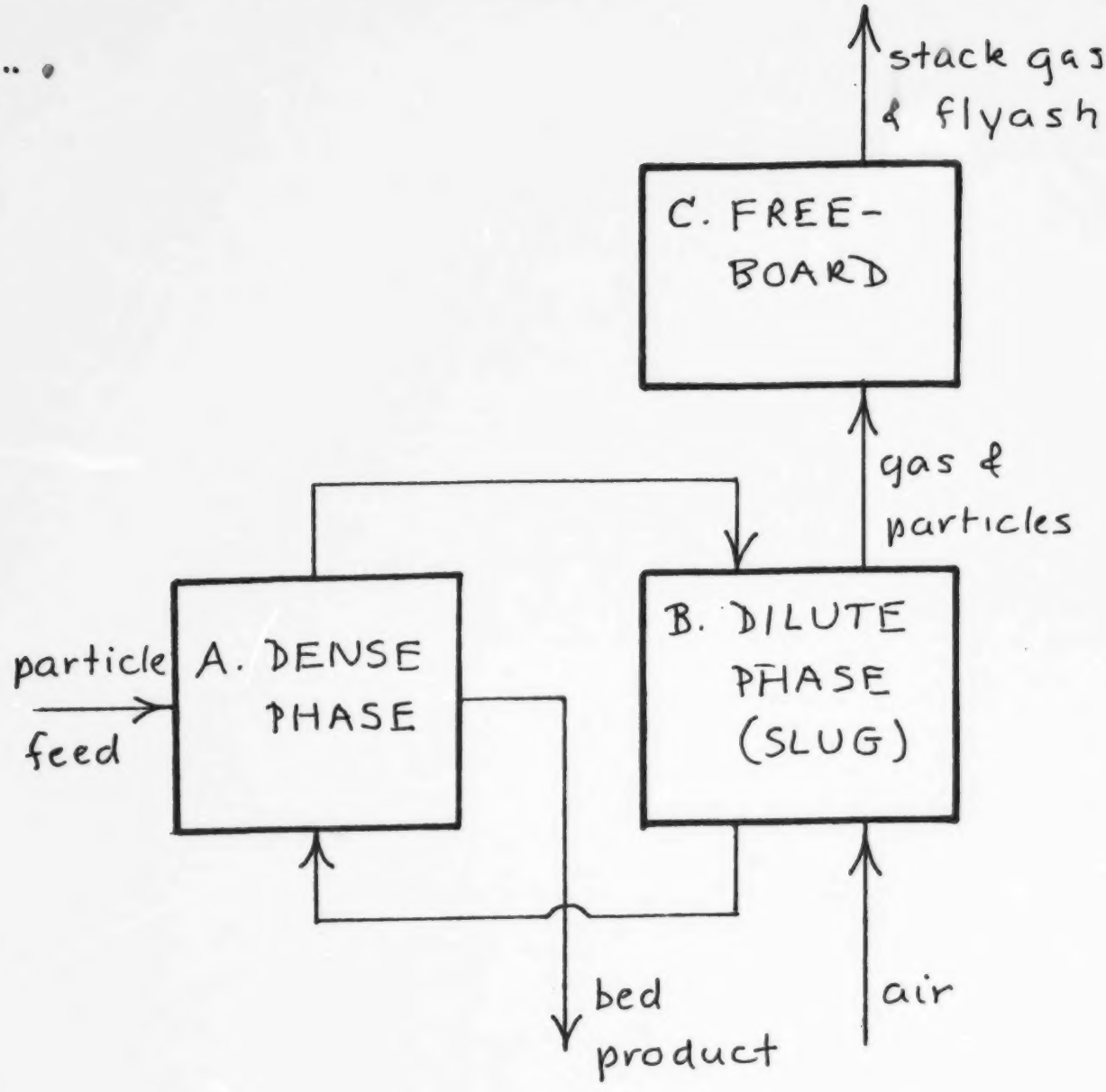


Fig. E.2. 3-compartment representation of system